Wiskunde Anibrand

Antwoordboek Graad 12

Annie Bothma

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Graad 12 Notaboek opgestel deur A Bothma

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Hoofstuk 1
Differensiaalrekene

Antwoorde 1: Gemiddelde helling tussen 2 punte

1.1 $f(x) = 2(x-1)^2 - 8; \quad x_A = 1 \quad en \quad x_B = 3$

A: $f(1) = 2(1-1)^2 - 8 = 2(0)^2 - 8 = -8$

B; $f(3) = 2(3-1)^2 - 8 = 2(2)^2 - 8 = 0$

$A(1;-8) \qquad B(3;0)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(3)-f(1)}{3-1} = \frac{0+8}{3-1} = \frac{8}{2} = 4$

1.2 $f(x) = -x^2 + 2x + 3 \quad x_A = -4 \quad en \quad x_B = -2$

A: $f(-4) = -(-4)^2 + 2(-4) + 3 = -16 - 8 + 3 = -21$

B: $f(-2) = -(-2)^2 + 2(-2) + 3 = -4 - 4 + 3 = -5$

$A(-4;-21) \qquad B(-2;-5)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(-4)-f(-2)}{-4+2} = \frac{-21+5}{-4+2} = \frac{-16}{-2} = 8$

1.3 $f(x) = \frac{5}{x-3} + 1 \quad x_A = -2 \quad en \quad x_B = 1$

A: $f(-2) = \frac{5}{-2-3} + 1 = \frac{5}{-5} + 1 = -1 + 1 = 0$

B; $f(1) = \frac{5}{1-3} + 1 = \frac{5}{-2} + 1 = -\frac{3}{2}$

$A(-2;0) \qquad B\left(1;-\frac{3}{2}\right)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(-2)-f(1)}{-2-1} = \frac{0+\frac{3}{2}}{-2-1} = \frac{\frac{3}{2}}{-\frac{3}{1}} = \frac{3}{2} \times \frac{1}{-3} = -\frac{1}{2}$

1.4 $f(x) = 3.2^{x+3} - 10 \quad x_A = -1 \quad en \quad x_B = 2$

A: $f(-1) = 3.2^{-1+3} - 10 = 3.2^2 - 10 = 12 - 10 = 2$

B; $f(2) = 3.2^{2+3} - 10 = 3.2^5 - 10 = 3.32 - 10 = 86$

$A(-1;2) \qquad B(2;86)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(-1)-f(2)}{-1-2} = \frac{2-86}{-1-2} = \frac{-84}{-3} = 28$

2. Onthou dat die gemiddelde spoed van 'n voorwerp gegee word deur $\frac{\Delta s}{\Delta t}$ en indien 'n grafiek van afstand teen tyd vir hierdie funksie geskets word, sal die gemiddelde helling van daardie grafiek tussen die punte $t = 2$ en $t = 3$ die spoed van die voorwerp tussen hierdie 2 tydstippe gee.

$s(t) = 5t^2$ vir $\quad t_1 = 2$ en $t_2 = 3$

Punt 1: $s(2) = 5t^2 = 5(2)^2 = 20m$

Punt 2: $s(3) = 5t^2 = 5(3)^2 = 45m$

gem spoed = $\frac{s(t+h)-s(t)}{h} = \frac{s(3)-s(2)}{3-2} = \frac{45-20}{1} = 25m.s^{-1} = 25m/s$

Hoofstuk 1
Differensiaalrekene

Antwoorde 2: Limiete

1. $\lim_{x\to 1} 2x = 2$

2. $\lim_{x\to 1} (2+x) = 2+1 = 3$

3. $\lim_{x\to 1} (2+x)^2 = (2+1)^2 = 3^2 = 9$

4. $\lim_{x\to 0} (x^2+3x+2) = (0)^2+3(0)+2 = 2$

5. $\lim_{x\to\infty} \frac{1}{2x} = 0$ onthou as jy ÷ met 'n baie groot getal, word die antwoord baie klein

6. $\lim_{x\to\infty} \frac{3x}{x+1} = \lim_{x\to\infty} \frac{\frac{3x}{x}}{\frac{x}{x}+\frac{1}{x}} = \lim_{x\to\infty} \frac{3}{1+\frac{1}{x}} = \frac{3}{1} = 3$ onthou $\lim_{x\to\infty} \frac{1}{x} = 0$

7. $\lim_{x\to\infty} \frac{x^2-3x+1}{3x^2-x+2} = \lim_{x\to\infty} \frac{\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{1}{x^2}}{\frac{3x^2}{x^2}-\frac{x}{x^2}+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{3}{x}+\frac{1}{x^2}}{3-\frac{1}{x}+\frac{2}{x^2}} = \frac{1}{3}$ onthou $\lim_{x\to\infty} \frac{1}{x} = 0$

8. $\lim_{x\to 0} \frac{4}{x} = \infty$ onthou as jy ÷ met 'n baie klein getal, word die antwoord baie groot

9. $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x\to 3} (x+3) = 3+3 = 6$

10. $\lim_{x\to 2} \frac{x^3-8}{x-2}$

$= \lim_{x\to 2} \frac{(x-2)\left(x^2+2x+4\right)}{(x-2)}$

$= \lim_{x\to 2} (x^2+2x+4)$

$= (2)^2+2(2)+4 = 12$

11. $\lim_{x\to 1} \frac{x^2+3x-4}{x^2-1} = \lim_{x\to 1} \frac{(x+4)(x-1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{(x+4)}{(x+1)} = \frac{1+4}{1+1} = \frac{5}{2}$

12. $\lim_{x\to 1} \frac{(x-1)(x+2)}{2x-2} = \lim_{x\to 1} \frac{(x-1)(x+2)}{2(x-1)} = \lim_{x\to 1} \frac{x+2}{2} = \frac{1+2}{2} = \frac{3}{2}$

13. $\lim_{x\to 0} \frac{x^2-4}{x-4} = \frac{0-4}{0-4} = \frac{-4}{-4} = 1$

14. $\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{(x-2)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$

15. $\lim_{x\to -1} \frac{2x+1}{x} = \frac{2(-1)+1}{(-1)} = \frac{-1}{-1} = 1$

16. $\lim_{x\to -1} \frac{x^2-3x-4}{x^2-1} = \lim_{x\to -1} \frac{(x+1)(x-4)}{(x-1)(x+1)} = \lim_{x\to -1} \frac{x-4}{x-1} = \frac{-1-4}{-1-1} = \frac{-5}{-2} = \frac{5}{2}$

Hoofstuk 1
Differensiaalrekene

Antwoorde 3: Helling van 'n punt op 'n kromme

1.1 $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ $\quad f(x) = 4$ dus $f(x+h) = 4$

$= \lim_{h \to 0} \frac{4-4}{h}$

$= \lim_{h \to 0} \frac{0}{h}$

$= \lim_{h \to 0} 0$

$= 0$

1.2 $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ $\quad f(x) = 3x^2$ dus $f(x+h) = 3(x+h)^2$

$= \lim_{h \to 0} \frac{3x^2+6hx+3h^2-3x^2}{h}$ $\quad = 3(x^2 + 2hx + h^2)$

$= \lim_{h \to 0} \frac{3h^2+6xh}{h}$ $\quad = 3h^2 + 6hx + 3x^2$

$= \lim_{h \to 0} \frac{h(3h+6x)}{h}$

$= \lim_{h \to 0} 3h + 6x$

$= 6x$

1.3 $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ $\quad f(x) = 3x + 1$ dus $f(x+h) = 3(x+h) + 1$

$= \lim_{h \to 0} \frac{3h+3x+1-(3x+1)}{h}$ $\quad = 3h + 3x + 1$

$= \lim_{h \to 0} \frac{3h+3x+1-3x-1}{h}$

$= \lim_{h \to 0} \frac{3h}{h}$

$= \lim_{h \to 0} 3$

$= 3$

1.4 $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ $\quad f(x) = 2x^3$ dus $f(x+h) = 2(x+h)^3$

$= \lim_{h \to 0} \frac{2h^3+6h^2x+6hx^2+2x^3-2x^3}{h}$ $\quad = 2(x+h)(x+h)^2$

$= \lim_{h \to 0} \frac{2h^3+6h^2x+6hx^2}{h}$ $\quad = 2(x+h)(x^2 + 2hx + h^2)$

$= \lim_{h \to 0} \frac{h\left(2h^2+6hx+6x^2\right)}{h}$ $\quad = 2(x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3)$

$= \lim_{h \to 0} 2h^2 + 6hx + 6x^2$ $\quad = 2(x^3 + 3hx^2 + 3h^2x + h^3)$

$= 6x^2$ $\quad = 2h^3 + 6h^2x + 6hx^2 + 2x^3$

1.5 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = x^2 + 2x - 1$ dus

$f(x+h) = (x+h)^2 + 2(x+h) - 1$

$= x^2 + 2hx + h^2 + 2x + 2h - 1$

$= \lim_{h\to 0} \frac{h^2+2hx+x^2+2h+2x-1-\left(x^2+2x-1\right)}{h}$

$= \lim_{h\to 0} \frac{h^2+2hx+x^2+2h+2x-1-x^2-2x+1}{h}$

$= \lim_{h\to 0} \frac{2h+2hx+h^2}{h}$

$= \lim_{h\to 0} \frac{h(h+2x+2)}{h}$

$= \lim_{h\to 0} h + 2x + 2$

$= 2x + 2$

1.6 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = -x^2 + x + 3$ dus

$f(x+h) = -(x+h)^2 + (x+h) + 3$

$= -(x^2 + 2hx + h^2) + x + h + 3$

$= -x^2 - 2hx - h^2 + x + h + 3$

$= \lim_{h\to 0} \frac{-x^2-2hx-h^2+x+h+3-\left(-x^2+x+3\right)}{h}$

$= \lim_{h\to 0} \frac{-x^2-2hx-h^2+x+h+3+x^2-x-3}{h}$

$= \lim_{h\to 0} \frac{h-2hx-h^2}{h}$

$= \lim_{h\to 0} \frac{h(1-2x-h)}{h}$

$= \lim_{h\to 0} 1 - 2x - h$

$= 1 - 2x$

1.7 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = \frac{2}{x}$ dus $f(x+h) = \frac{2}{x+h}$

$= \lim_{h\to 0} \frac{\frac{2}{x+h}-\frac{2}{x}}{h}$

$= \lim_{h\to 0} \frac{\frac{2x-2(x+h)}{x(x+h)}}{h}$

$= \lim_{h\to 0} \frac{\frac{2x-2x-2h}{x^2+xh}}{h}$

$= \lim_{h\to 0} \frac{\frac{-2h}{x^2+xh}}{\frac{h}{1}}$

$= \lim_{h\to 0} \frac{-2h}{x^2+xh} \times \frac{1}{h}$

$= \lim_{h\to 0} \frac{-2}{x^2+hx}$

$= \frac{-2}{x^2}$

1.8 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = nx^2$ dus $f(x+h) = n(x+h)^2$

$= n(x^2 + 2hx + h^2)$

$= nx^2 + 2nhx + nh^2$

$= \lim_{h\to 0} \frac{nx^2+2nhx+nh^2-nx^2}{h}$

$= \lim_{h\to 0} \frac{2nhx+nh^2}{h}$

$= \lim_{h\to 0} \frac{h(2nx+nh)}{h}$

$= \lim_{h\to 0} 2nx + nh$

$= 2nx$

2.1 $f(x) = \frac{1}{2}x^2$ by $x = -1$ $\quad$ $f(x) = \frac{1}{2}x^2$ dus $f(x+h) = \frac{1}{2}(x+h)^2$

$= \frac{1}{2}\left(x^2 + \frac{2}{1}hx + h^2\right)$

$= \frac{1}{2}x^2 + hx + \frac{1}{2}h^2$

$f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2}x^2+hx+\frac{1}{2}h^2-\left(\frac{1}{2}x^2\right)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2}x^2+hx+\frac{1}{2}h^2-\frac{1}{2}x^2}{h}$

$= \lim_{h\to 0} \frac{hx+\frac{1}{2}h^2}{h}$

$= \lim_{h\to 0} \frac{h\left(x+\frac{1}{2}h\right)}{h}$

$= \lim_{h\to 0} x + \frac{1}{2}h$

$f'(x) = x$

$f'(-1) = -1$

2.2 $f(x) = -x^2 + 1$ by $x = 1$ $\quad$ $f(x) = -x^2 + 1$

$f(x+h) = -(x+h)^2 + 1$

$= -(x^2 + 2hx + h^2) + 1$

$= -x^2 - 2hx - h^2 + 1$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{-x^2-2hx-h^2+1-\left(-x^2+1\right)}{h}$

$= \lim_{h\to 0} \frac{-x^2-2hx-h^2+1+x^2-1}{h}$

$= \lim_{h\to 0} \frac{-2hx-h^2}{h}$

$= \lim_{h\to 0} \frac{h(-2x-h)}{h}$

$= \lim_{h\to 0} -2x - h$

$f'(x) = -2x$

$f'(1) = -2(1) = -2$

2.3 $f(x) = 2(x^2 - 1)$ by $x = 3$ $\quad$ $f(x) = 2(x^2 - 1) = 2x^2 - 2$

$f(x+h) = 2(x+h)^2 - 2$

$= 2(x^2 + 2hx + h^2) - 2$

$= 2x^2 + 4hx + 2h^2 - 2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{2x^2+4hx+2h^2-2-\left(2x^2-2\right)}{h}$

$= \lim_{h\to 0} \frac{2x^2+4hx+2h^2-2-2x^2+2}{h}$

$= \lim_{h\to 0} \frac{4hx+2h^2}{h}$

$= \lim_{h\to 0} \frac{h(4x+2h)}{h}$

$= \lim_{h\to 0} 4x + 2h$

$f'(x) = 4x$

$f'(3) = 4(3) = 12$

2.4 $f(x) = 3x^2 - 4$ by $x = 0$ $\quad$ $f(x) = 3x^2 - 4$

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$f(x+h) = 3(x+h)^2 - 4$

$= 3(x^2 + 2hx + h^2) - 4$

$= 3x^2 + 6hx + 3h^2 - 4$

$= \lim_{h \to 0} \frac{3x^2+6hx+3h^2-4-\left(3x^2-4\right)}{h}$

$= \lim_{h \to 0} \frac{3x^2+6hx+3h^2-4-3x^2+4}{h}$

$= \lim_{h \to 0} \frac{6hx+3h^2}{h}$

$= \lim_{h \to 0} \frac{h(6x+3h)}{h}$

$= \lim_{h \to 0} 6x + 3h$

$f'(x) = 6x$

$f'(0) = 6(0) = 0$

3.1 $\frac{f(-1+h)-f(-1)}{h}$

$f(x) = 2x^3$ dus $f(-1+h) = 2(-1+h)^3$

$= 2(-1+h)(-1+h)^2$

$= 2(-1+h)(1-2h+h^2)$

$= 2(-1+2h-h^2+h-2h^2+h^3)$

$= 2(-1+3h-3h^2+h^3)$

$= -2+6h-6h^2+2h^3$

$f(-1) = 2(-1)^3 = 2(-1) = -2$

$= \frac{-2+6h-6h^2+2h^3-(-2)}{h}$

$= \frac{-2+6h-6h^2+2h^3+2}{h}$

$= \frac{6h-6h^2+2h^3}{h}$

$= \frac{h\left(6-6h+2h^2\right)}{h}$

$= 6 - 6h + 2h^2$

3.2 $\lim_{h \to 0} \frac{f(-1+h)-f(-1)}{h}$

$= \lim_{h \to 0} 6 - 6h + 2h^2$

$f'(-1) = 6$

3.3 Die helling van grafiek f by $x = -1$ is 6

4. Gegee: $p'(a)$ se antwoord is $\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h}$

Onthou: $p'(x) = \lim_{h \to 0} \frac{p(x+h)-p(x)}{h}$

$\therefore p'(a) = \lim_{h \to 0} \frac{p(a+h)-p(a)}{h}$

$= \lim_{h \to 0} \frac{\sqrt{9+h}-3}{h}$

Met inspeksie volg dan dat $p(a+h) = \sqrt{9+h}$ en dus moet $p(a) = \sqrt{9} = 3$

$\therefore$ was $p(x+h) = \sqrt{x+h}$ en $p(x) = \sqrt{x}$ waar x toe vervang is met 9

Dit volg dus dat $p(x) = \sqrt{x}$ en dat $a = 9$

Hoofstuk 1
Differensiaalrekene

Antwoorde 4: Differensiasie reëls

1.1 $f(x) = x$

$f'(x) = 1$

1.2 $f(x) = 4x$

$f'(x) = 4$

1.3 $f(x) = 4x^2$

$f'(x) = 8x$

1.4 $f(x) = 4x^3$

$f'(x) = 12x^2$

1.5 $f(x) = 4$

$f'(x) = 0$

1.6 $f(x) = -4x$

$f'(x) = -4$

1.7 $f(x) = 4 - x$

$f'(x) = -1$

1.8 $f(x) = 4x^{-1}$

$f'(x) = -4x^{-2} = \frac{-4}{x^2}$

1.9 $f(x) = 4 - \frac{1}{x}$

$f(x) = 4 - 1.x^{-1}$

$f'(x) = x^{-2} = \frac{1}{x^2}$

1.10 $f(x) = 4x - \frac{4}{x}$

$f(x) = 4x - 4.x^{-1}$

$f'(x) = 4 + 4x^{-2} = 4 + \frac{4}{x^2}$

2.1 $y = x^{-2}$

$\frac{dy}{dx} = -2x^{-3} = \frac{-2}{x^3}$

2.2 $y = x^{100}$

$\frac{dy}{dx} = 100x^{99}$

2.3 $y = x^{0,3}$

$\frac{dy}{dx} = 0,3x^{0,3-1} = 0,3x^{-0,7} = \frac{0,3}{x^{0,7}}$

2.4 $y = x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

2.5 $y = 3x^{-3}$

$\frac{dy}{dx} = 3.(-3)x^{-4} = \frac{-9}{x^4}$

2.6 $y = 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2$

2.7 $y = \frac{1}{2x^2}$

$y = \frac{1.x^{-2}}{2}$

$\frac{dy}{dx} = \frac{-2x^{-3}}{2} = \frac{-1}{x^3}$

2.8 $x^2y = 2$

$y = \frac{2}{x^2} = 2.x^{-2}$

$\frac{dy}{dx} = -4x^{-3} = \frac{-4}{x^3}$

2.9 $y = \sqrt[3]{8x}$

$y = 2x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{2}{1}.\frac{1}{3}x^{\frac{1}{3}-1}$

$= \frac{2}{3}x^{-\frac{2}{3}} = \frac{2}{3x^{\frac{2}{3}}} = \frac{2}{3\sqrt[3]{x^2}}$

2.10 $y = 2\sqrt{x}$

$y = 2x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{2}{1}.\frac{1}{2}x^{\frac{1}{2}-1}$

$= x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$

3.1 $\frac{d}{dx}(2x - 3)$

$= 2$

3.2 $\frac{d}{dx}(4x + 5)$

$= 4$

3.3 $\frac{d}{dx}[x^4 - 3x^2]$

$= 4x^3 - 6x$

3.4 $\frac{d}{dx}[mx + c]$

$= m$

3.5 $\frac{d}{dx}[ax^3 + bx^2 + cx + d]$

$= 3ax^2 + 2bx + c$

3.6 $\frac{d}{dx}[(x + 2)(x - 4)]$

$= \frac{d}{dx}[x^2 - 2x - 8]$

3.7 $\frac{d}{dx}[(x^2-3)(x+1)]$

$= \frac{d}{dx}[x^3+x^2-3x-3]$

$= 3x^2+2x-3$

3.9 $\frac{d}{dx}\left[\frac{4x^2-1}{2x+1}\right]$

$= \frac{d}{dx}\left[\frac{(2x-1)(2x+1)}{(2x+1)}\right]$

$= \frac{d}{dx}[2x-1]$

$= 2$

4.1 $D_x[x^2-3x+2]$

$= 2x-3$

4.2 $D_x[8x^3+3x^2-11]$

$= 24x^2+6x$

4.4 $D_x\left[\sqrt{x^2-6x+9}\right]$

$= D_x\left[\sqrt{(x-3)(x-3)}\right]$

$= D_x\left[\sqrt{(x-3)^2}\right]$

$= D_x[x-3]$

$= 1$

4.5 $D_x\left[\frac{x^2-3x-4}{x+1}\right]$

$= D_x\left[\frac{(x+1)(x-4)}{(x+1)}\right]$

$= D_x[x-4]$

$= 1$

4.7 $D_x\left[5\sqrt{x}-\sqrt{5x}\right]$

$= D_x\left[5x^{\frac{1}{2}}-\sqrt{5}\,x^{\frac{1}{2}}\right]$

$= \frac{5}{1}.\frac{1}{2}x^{-\frac{1}{2}}-\frac{\sqrt{5}}{1}.\frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{5}{2x^{\frac{1}{2}}}-\frac{\sqrt{5}}{2x^{\frac{1}{2}}} = \frac{5}{2\sqrt{x}}-\frac{\sqrt{5}}{2\sqrt{x}}$

4.9 $D_x\left[\frac{x^2-3x-4}{x}\right]$

$= D_x\left[\frac{x^2}{x}-\frac{3x}{x}-\frac{4}{x}\right]$

$= D_x\left[x-3-\frac{4}{x}\right]$

$= D_x[x-3-4.x^{-1}]$

$= 1+4x^{-2} = 1+\frac{4}{x^2}$

$= 2x-2$

3.5 $\frac{d}{dx}\left[(2x-1)^2\right]$

$= \frac{d}{dx}[4x^2-4x+1]$

$= 8x-4$

3.10 $\frac{d}{dx}\left[\sqrt{x}-\frac{1}{\sqrt{x}}\right]$

$= \frac{d}{dx}\left[x^{\frac{1}{2}}-\frac{1}{x^{\frac{1}{2}}}\right]$

$= \frac{d}{dx}\left[x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right]$

$= \frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2x^{\frac{1}{2}}}+\frac{1}{2x^{\frac{3}{2}}}$

$= \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{x^3}}$

4.3 $D_x[3x^3-4x^2+2x+1]$

$= 9x^2-8x+2$

4.6 $D_x\left[\frac{1}{x^2}-\frac{1}{x}\right]$

$= D_x[1.x^{-2}-1.x^{-1}]$

$= -2x^{-3}+x^{-2}$

$= \frac{-2}{x^3}+\frac{1}{x^2}$

4.8 $D_x\left[\frac{3x^3-4x^4}{x^4}\right]$

$= D_x\left[\frac{3x^3}{x^4}-\frac{4x^4}{x^4}\right]$

$= D_x\left[\frac{3}{x}-4\right]$

$= D_x[3.x^{-1}-4]$

$= -3x^{-2} = \frac{-3}{x^2}$

4.10 $D_x\left[\sqrt{x^3}-\sqrt[3]{x^2}\right]$

$= D_x\left[x^{\frac{3}{2}}-x^{\frac{2}{3}}\right]$

$= \frac{3}{2}x^{\frac{3}{2}-1}-\frac{2}{3}x^{\frac{2}{3}-1}$

$= \frac{3}{2}x^{\frac{1}{2}}-\frac{2}{3}x^{-\frac{1}{3}}$

$= \frac{3}{2}x^{\frac{1}{2}}-\frac{2}{3x^{\frac{1}{3}}}$

$= \frac{3}{2}\sqrt{x}-\frac{2}{3\sqrt[3]{x}}$

5. $xy + y = x^2 - 1$

$y(x+1) = x^2 - 1$

$\frac{y(x+1)}{(x+1)} = \frac{x^2-1}{(x+1)}$

$y = \frac{(x-1)(x+1)}{(x+1)}$

$y = x - 1$

$\frac{dy}{dx} = 1$

6. $4t^2 - 2tr + r - 1 = 0$

$-2tr + r = 1 - 4t^2$

$r(-2t+1) = 1 - 4t^2$

$\frac{r(-2t+1)}{(-2t+1)} = \frac{1-4t^2}{(-2t+1)}$

$r = \frac{(1-2t)(1+2t)}{(-2t+1)}$

$r = 1 + 2t$

$\frac{dr}{dt} = 2$

7. $f(y) = \frac{16}{y} = 16y^{-1}$

$f'(y) = -16y^{-2}$

$= \frac{-16}{y^2}$

$4f'(y) = \frac{4}{1}\left(\frac{-16}{y^2}\right) = \frac{-64}{y^2}$

$f'(4y) = \frac{-16}{(4y)^2}$

$= \frac{-16}{16y^2}$

$= \frac{-1}{y^2}$

$\therefore f'(4y) - 4f'(y) = \frac{-1}{y^2} - \frac{-64}{y^2}$

$= \frac{-1+64}{y^2} = \frac{63}{y^2}$

8. $y = 2x(3-x)$ en $z = \frac{y}{2}$

$y = 6x - 2x^2$

8.1 $\frac{dy}{dx} = 6 - 4x$

8.2 $\frac{dz}{dy} = \frac{1}{2}$

8.3 $y = 6x - 2x^2 \ldots.(1)$ en $\frac{z}{1} = \frac{y}{2}$

Stel y in (1) $\qquad 2z = y \ldots..(2)$

$2z = 6x - 2x^2$

$\frac{2z}{2} = \frac{6x}{2} - \frac{2x^2}{2}$

$z = 3x - x^2$

$\frac{dz}{dx} = 3 - 2x$

9. $\sqrt{x} = y^{\frac{1}{3}}$ en $y = z^{-3}$

9.1 $\sqrt{x} = \sqrt[3]{y}$

$\sqrt{x}^3 = \left(\sqrt[3]{y}\right)^3$

$x^{\frac{3}{2}} = y$

$y = x^{\frac{3}{2}}$

$\therefore \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3}{2}\sqrt{x}$

9.2 $y = x^{\frac{3}{2}} \ldots..(1)$ en $y = z^{-3} \ldots.(2)$

stel y in (1)

$z^{-3} = x^{\frac{3}{2}}$

$z^{-3} = \sqrt{x^3}$

$\left(z^{-3}\right)^2 = \left(\sqrt{x^3}\right)^2$

$z^{-6} = x^3$

$\sqrt[3]{z^{-6}} = \sqrt[3]{x^3}$

$z^{-2} = x$

$x = z^{-2}$

$\frac{dx}{dz} = -2z^{-3}$

$= \frac{-2}{z^3}$

9.3 $y = z^{-3}$

$\frac{dy}{dz} = -3z^{-4}$

$= \frac{-3}{z^4}$

Hoofstuk 1
Differensiaalrekene

Antwoorde 5a: Vergelyking van 'n raaklyn aan 'n kromme

1.Bereken die vergelyking van die raaklyn in elk van die volgende gevalle:

1.1 $f(x) = x^2 - 2x + 1$ by $x = -1$

RP $(-1;y)$

$f(-1) = (-1)^2 - 2(-1) + 1 = 1 + 2 + 1 = 4$

RP $(-1;4)$

$f'(x) = 2x - 2$

$f'(-1) = 2(-1) - 2 = -2 - 2 = -4$

$m_{raaklyn} = f'(-1) = -4$ en RP$(-1;4)$

Raaklyn: $y - y_1 = m(x - x_1)$

$y - 4 = -4(x + 1)$

$y - 4 = -4x - 4$

$y = -4x$

1.2 $f(x) = x^3 - 2x^2 - 9x + 18$ by $x = 1$

RP $(1;y)$

$f(1) = (1)^3 - 2(1)^2 - 9(1) + 18 = 1 - 2 - 9 + 18 = 8$

RP $(1;8)$

$f'(x) = 3x^2 - 4x - 9$

$f'(1) = 3(1)^2 - 4(1) - 9 = 3 - 4 - 9 = -10$

$m_{raaklyn} = f'(1) = -10$ en RP$(1;8)$

Raaklyn: $y - y_1 = m(x - x_1)$

$y - 8 = -10(x - 1)$

$y - 8 = -10x + 10$

$y = -10x + 18$

1.3 $g(x) = 4\sqrt{x}$ by $x = 4$

RP $(4;y)$

$g(4) = 4\sqrt{4} = 4(2) = 8$

RP $(4;8)$

$g(x) = 4\sqrt{x} = 4x^{\frac{1}{2}}$

$g'(x) = \frac{4}{1}\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} = 2x^{-\frac{1}{2}} = \frac{2}{x^{\frac{1}{2}}} = \frac{2}{\sqrt{x}}$

$g'(4) = \frac{2}{\sqrt{4}} = \frac{2}{2} = 1$

$m_{raaklyn} = g'(4) = 1$ en RP$(4;8)$

Raaklyn: $y - y_1 = m(x - x_1)$

$y - 8 = 1(x - 4)$

$y - 8 = x - 4$

$y = x + 4$

1.4 $p(x) = \frac{3}{x}$ by $x = 1$

RP $(1;y)$

$p(1) = \frac{3}{1} = 3$

RP $(1;3)$

$p(x) = \frac{3}{x} = 3x^{-1}$

$p'(x) = -3x^{-2} = -\frac{3}{x^2}$

$p'(1) = -\frac{3}{(1)^2} = -\frac{3}{1} = -3$

$m_{raaklyn} = p'(1) = -3$ en RP$(1;3)$

Raaklyn: $y - y_1 = m(x - x_1)$

$y - 3 = -3(x - 1)$

$y - 3 = -3x + 3$

$y = -3x + 6$

2.1 $f(x) = 2x^2 - 3x$ en $m_{raaklyn} = f'(x_{RP}) = 5$

$f'(x) = 4x - 3$ $f(2) = 2(2)^2 - 3(2)$

$5 = 4x - 3$ $= 8 - 6$

$8 = 4x$ $= 2$

$2 = x$ punt is $(2;2)$

2.2 $f(x) = 2x^2 - 3x$ en $m_{raaklyn} = f'(x) = -1$

$f'(x) = 4x - 3$ $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right)$

$-1 = 4x - 3$ $= \frac{2}{1}\left(\frac{1}{4}\right) - \frac{3}{1}\left(\frac{1}{2}\right)$

$2 = 4x$ $= \frac{1}{2} - \frac{3}{2} = -1$

$\frac{1}{2} = x$ punt is $\left(\frac{1}{2};-1\right)$

3.

Stel AB: $y = -2x + 1$ en $f(x) = x^2 + 2x - 3$

$m_{AB} = -2$

$m_{raaklyn} = m_{AB} = -2$ raaklyn $//AB$

$m_{raaklyn} = f'(x) = -2$

$f'(x) = 2x + 2$ $f(-2) = (-2)^2 + 2(-2) - 3$

$-2 = 2x + 2$ $= 4 - 4 - 3$

$-4 = 2x$ $= -3$

$-2 = x$ punt is $(-2;-3)$

4.

Stel AB: $y = x$ en $f(x) = 4 - x^2$

$m_{AB} = 1$

$m_{raaklyn} = -1$ raaklyn $\perp AB$

$m_{raaklyn} = f'(x) = -1$

$f'(x) = -2x$

$-1 = -2x$

$\frac{1}{2} = x$

$f\left(\frac{1}{2}\right) = 4 - \left(\frac{1}{2}\right)^2$

$= 4 - \frac{1}{4}$

$= 3\frac{3}{4}$

punt is $\left(\frac{1}{2}; 3\frac{3}{4}\right)$

5. $m_{raaklyn} = \tan 45°$ Die raaklyn se inklinasiehoek is $45°$

$\therefore m_{raaklyn} = 1$

$f(x) = 12x^3$

$\therefore f'(x) = 36x^2$ formule om helling by enige x-waarde te bereken

maar $f'(x) = m_{raaklyn} = 1$ helling van punt is dieselfde as helling van die raaklyn

$1 = 36x^2$

$\frac{1}{36} = x^2$

$\pm\sqrt{\frac{1}{36}} = \sqrt{x^2}$

$\pm\frac{1}{6} = x$

Hoofstuk 1
Differensiaalrekene

Antwoorde 5b: Gemeng deel 1

1.1 $\lim_{h\to 0} \frac{4(1+h)^2-4}{h}$

$=\lim_{h\to 0} \frac{4(1+2h+h^2)-4}{h}$

$=\lim_{h\to 0} \frac{4+8h+4h^2-4}{h}$

$=\lim_{h\to 0} \frac{8h+4h^2}{h}$

$=\lim_{h\to 0} \frac{h(8+4h)}{h}$

$=\lim_{h\to 0} 8+4h$

$= 8$

1.2 $\lim_{h\to 0} \frac{3(x+h)^2-3x^2}{h}$

$=\lim_{h\to 0} \frac{3(x^2+2hx+h^2)-3x^2}{h}$

$=\lim_{h\to 0} \frac{3x^2+6hx+3h^2-3x^2}{h}$

$=\lim_{h\to 0} \frac{6hx+3h^2}{h}$

$=\lim_{h\to 0} \frac{h(6x+3h)}{h}$

$=\lim_{h\to 0} 6x+3h$

$= 6x$

1.3 $\lim_{x\to -\frac{1}{2}} \frac{2x^2-x-1}{2x+1} = \lim_{x\to -\frac{1}{2}} \frac{(2x+1)(x-1)}{(2x+1)} = \lim_{x\to -\frac{1}{2}} x-1 = -\frac{1}{2}-1 = -1\frac{1}{2}$

$g(x) = \frac{-5}{x}$

$g(x+h) = \frac{-5}{x+h}$

2.1 $g'(x) =\lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$=\lim_{h\to 0} \frac{\frac{-5}{x+h}-\left(\frac{-5}{x}\right)}{h}$

$=\lim_{h\to 0} \frac{\frac{-5}{x+h}+\frac{5}{x}}{h}$

$=\lim_{h\to 0} \frac{\frac{-5x+5(x+h)}{x(x+h)}}{\frac{h}{1}}$

$=\lim_{h\to 0} \frac{-5x+5x+5h}{x^2+hx} \times \frac{1}{h}$

$=\lim_{h\to 0} \frac{5h}{x^2+hx} \times \frac{1}{h}$

$=\lim_{h\to 0} \frac{5}{x^2+hx}$

$= \frac{5}{x^2}$

2.2 $g(x) = \frac{-5}{x}$

$g(-1) = \frac{-5}{-1} = 5$

$RP(-1;5)$

$g'(x) = \frac{5}{x^2}$

$g'(-1) = \frac{5}{(-1)^2} = \frac{5}{1} = 5$

$m_{raaklyn} = 5$ en $(-1;5)$

Raaklyn:

$y-y_1 = m(x-x_1)$

$y-5 = 5(x+1)$

$y-5 = 5x+5$

$y = 5x+10$

3.1 $\frac{d}{dx}(3x^2-7x-6)(x-3)^{-1}$

$= \frac{d}{dx}\frac{(3x^2-7x-6)}{(x-3)^1}$

$= \frac{d}{dx}\frac{(3x+2)(x-3)}{(x-3)}$

$= \frac{d}{dx}(3x+2)$

$= 3$

3.2 $D_x\left(6\sqrt[3]{x^2}\right)$

$= D_x\left(6x^{\frac{2}{3}}\right)$

$= \frac{6}{1}.\frac{2}{3}x^{\frac{2}{3}-1}$

$= 4x^{-\frac{1}{3}}$

$= \frac{4}{x^{\frac{1}{3}}}$

$= \frac{4}{\sqrt[3]{x}}$

3.3 $h(x) = \left(x^3 - \frac{3}{x}\right)\left(x^3 + \frac{3}{x}\right)$

$= x^6 - \frac{9}{x^2}$

$= x^6 - 9x^{-2}$

$h'(x) = 6x^5 + 18x^{-3}$

$= 6x^5 + \frac{18}{x^3}$

3.4 $y = \frac{x^3-x^2-2x}{x+1}$

$= \frac{x\left(x^2-x-2\right)}{x+1}$

$= \frac{x(x+1)(x-2)}{(x+1)}$

$= x(x-2)$

$= x^2 - 2x$

$\frac{dy}{dx} = 2x - 2$

3.5 $D_x\left(\sqrt{x} + \frac{6}{\sqrt[3]{x^2}} + \pi x^3\right)$

$= D_x\left(x^{\frac{1}{2}} + \frac{6}{x^{\frac{2}{3}}} + \pi x^3\right)$

$= D_x\left(x^{\frac{1}{2}} + 6x^{-\frac{2}{3}} + \pi x^3\right)$

$= \frac{1}{2}x^{-\frac{1}{2}} + \frac{6}{1}.\left(-\frac{2}{3}\right)x^{-\frac{5}{3}} + 3\pi x^2$

$= \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-\frac{5}{3}} + 3\pi x^2$

$= \frac{1}{2x^{\frac{1}{2}}} - \frac{4}{x^{\frac{5}{3}}} + 3\pi x^2$

$= \frac{1}{2\sqrt{x}} - \frac{4}{\sqrt[3]{x^5}} + 3\pi x^2$

3.6 $g(x) = \frac{1-2x+\sqrt{x}}{x^2}$

$= \frac{1}{x^2} - \frac{2x}{x^2} + \frac{x^{\frac{1}{2}}}{x^2}$

$= \frac{1}{x^2} - \frac{2}{x} + x^{\frac{1}{2}}.x^{-2}$

$= x^{-2} - 2x^{-1} + x^{-\frac{3}{2}}$

$g'(x) = -2x^{-3} + 2x^{-2} - \frac{3}{2}x^{-\frac{5}{2}}$

$= -\frac{2}{x^3} + \frac{2}{x^2} - \frac{3}{2x^{\frac{5}{2}}}$

$= -\frac{2}{x^3} + \frac{2}{x^2} - \frac{3}{2\sqrt{x^5}}$

3.7 $D_x\left(\frac{\left(2x^2-5\right)(3x+2)}{x^2}\right)$

$= D_x\left(\frac{6x^3+4x^2-15x-10}{x^2}\right)$

$= D_x\left(\frac{6x^3}{x^2} + \frac{4x^2}{x^2} - \frac{15x}{x^2} - \frac{10}{x^2}\right)$

$= D_x\left(6x + 4 - \frac{15}{x} - \frac{10}{x^2}\right)$

$= D_x(6x + 4 - 15x^{-1} - 10x^{-2})$

$= 6 + 15x^{-2} + 20x^{-3}$

$= 6 + \frac{15}{x^2} + \frac{20}{x^3}$

3.8 $xy = 5$

$y = \frac{5}{x}$

$= 5x^{-1}$

$\frac{dy}{dx} = -5x^{-2}$

$= -\frac{5}{x^2}$

4.1 $f(x) = x^2 - 4x$

$f(-1) = (-1)^2 - 4(-1) = 5$

$f(1) = (1)^2 - 4(1) = -3$

gemiddelde helling $= \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-3-5}{1+1}$

$= \frac{-8}{2} = -4$

4.2 $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f'(-1) = 2(-1) - 4 = -6$

5. $y = -\frac{1}{2}x + 4$

$m = -\frac{1}{2}$

$m_{raaklyn} = 2$ lyne $\perp$ mekaar

$f'(x) = 2$

$y = f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f'(x) = m_{raaklyn} = 2$

$2 = 2x - 4$

$6 = 2x$

$3 = x$

die raaklyn raak f by $x = 3$

$f(x) = x^2 - 4x$

$f(3) = (3)^2 - 4(3) = -3$

$RP(3;-3)$ en $m_{raaklyn} = 2$

Raaklyn: $y - y_1 = m(x - x_1)$

$y + 3 = 2(x - 3)$

$y + 3 = 2x - 6$

$y = 2x - 9$

Meer oor "Wiskunde Anibrand Antwoordboek Graad 12" en die outeur.

Ek is reeds vir 28 jaar betrokke by Wiskunde-onderrig vir graad 8 tot graad 12 leerders. Die afgelope 10 jaar is ek verbonde aan Hoërskool Die Wilgers in Pretoria, waar ek 'n Wiskunde Akademie bedryf met een groep in elke graad.

Met die aanvang van die nuwe KABV sillabus in 2007 het ek begin om al my notas vir my Wiskunde-onderrig elektronies saam te stel met behulp van innoverende sagteware sodat dit alle onderwerpe met grafika en voorbeelde volledig verduidelik. Die graad 12 Wiskunde Anibrand Antwoordboek bied volledig uitgewerkte, verduidelikende antwoorde vir al die huiswerk probleme in die graad 12 "Wiskunde Anibrand Notaboek".

Leerders wat hierdie boek gebruik om hulle huiswerk probleme te merk, kan uit die antwoorde self sien waar hulle gefouteer het en dan hulle foute korrigeer.

Ek gebruik hierdie antwoorde die afgelope 5 jaar in my klasaanbieding vir die graad 12 leerders. Dit stel my in staat om die antwoorde konstant te verbeter, soos wat ek dit in die klassituasie as nodig ervaar.

Die volledige antwoorde op huiswerprobleme help leerders om selfvertroue in die vak te kry want hulle kan hulle probleme identifiseer wanneer hulle huiswerk doen, dit uitsorteer en dan voortgaan met die res van die huiswerk probleme.

Hierdie boek, saam met die Notaboek, is die antwoord vir alle graad 12 leerders wat wil presteer inWiskunde en ook vir alle Wiskunde onderwysers wat sonder moeite 'n kwaliteit Wiskunde klasaanbieding vir leerders wil bied.

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