

Wiskunde Anibrand

Antwoordboek Graad 12



Annie Bothma

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Annie Bothma

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Hoofstuk 1

Differensiaalrekenne

Antwoorde 1: Gemiddelde helling tussen 2 punte

1.1 $f(x) = 2(x-1)^2 - 8$; $x_A = 1$ en $x_B = 3$

A: $f(1) = 2(1-1)^2 - 8 = 2(0)^2 - 8 = -8$

B: $f(3) = 2(3-1)^2 - 8 = 2(2)^2 - 8 = 0$

$A(1; -8)$ $B(3; 0)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(3)-f(1)}{3-1} = \frac{0+8}{3-1} = \frac{8}{2} = 4$

1.2 $f(x) = -x^2 + 2x + 3$ $x_A = -4$ en $x_B = -2$

A: $f(-4) = -(-4)^2 + 2(-4) + 3 = -16 - 8 + 3 = -21$

B: $f(-2) = -(-2)^2 + 2(-2) + 3 = -4 - 4 + 3 = -5$

$A(-4; -21)$ $B(-2; -5)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(-4)-f(-2)}{-4+2} = \frac{-21+5}{-4+2} = \frac{-16}{-2} = 8$

1.3 $f(x) = \frac{5}{x-3} + 1$ $x_A = -2$ en $x_B = 1$

A: $f(-2) = \frac{5}{-2-3} + 1 = \frac{5}{-5} + 1 = -1 + 1 = 0$

B: $f(1) = \frac{5}{1-3} + 1 = \frac{5}{-2} + 1 = -\frac{3}{2}$

$A(-2; 0)$ $B(1; -\frac{3}{2})$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(-2)-f(1)}{-2-1} = \frac{0+\frac{3}{2}}{-2-1} = \frac{\frac{3}{2}}{-\frac{3}{1}} = \frac{3}{2} \times \frac{1}{-3} = -\frac{1}{2}$

1.4 $f(x) = 3 \cdot 2^{x+3} - 10$ $x_A = -1$ en $x_B = 2$

A: $f(-1) = 3 \cdot 2^{-1+3} - 10 = 3 \cdot 2^2 - 10 = 12 - 10 = 2$

B: $f(2) = 3 \cdot 2^{2+3} - 10 = 3 \cdot 2^5 - 10 = 3 \cdot 32 - 10 = 86$

$A(-1; 2)$ $B(2; 86)$

gem helling = $\frac{f(x+h)-f(x)}{h} = \frac{f(-1)-f(2)}{-1-2} = \frac{2-86}{-1-2} = \frac{-84}{-3} = 28$

2. Onthou dat die gemiddelde spoed van 'n voorwerp gegee word deur $\frac{\Delta s}{\Delta t}$ en indien 'n grafiek van afstand teen tyd vir hierdie funksie geskets word, sal die gemiddelde helling van daardie grafiek tussen die punte $t = 2$ en $t = 3$ die spoed van die voorwerp tussen hierdie 2 tydsteppe gee.

$s(t) = 5t^2$ vir $t_1 = 2$ en $t_2 = 3$

Punt 1: $s(2) = 5t^2 = 5(2)^2 = 20m$

Punt 2: $s(3) = 5t^2 = 5(3)^2 = 45m$

gem spoed = $\frac{s(t+h)-s(t)}{h} = \frac{s(3)-s(2)}{3-2} = \frac{45-20}{1} = 25m \cdot s^{-1} = 25m/s$

Hoofstuk 1

Differensiaalrekenne

Antwoorde 2: Limiete

$$1. \lim_{x \rightarrow 1} 2x = 2$$

$$2. \lim_{x \rightarrow 1} (2 + x) = 2 + 1 = 3$$

$$3. \lim_{x \rightarrow 1} (2 + x)^2 = (2 + 1)^2 = 3^2 = 9$$

$$4. \lim_{x \rightarrow 0} (x^2 + 3x + 2) = (0)^2 + 3(0) + 2 = 2$$

$$5. \lim_{x \rightarrow \infty} \frac{1}{2x} = 0 \quad \text{onthou as jy } \div \text{ met 'n baie groot getal, word die antwoord baie klein}$$

$$6. \lim_{x \rightarrow \infty} \frac{3x}{x+1} = \lim_{x \rightarrow \infty} \frac{\frac{3x}{x}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{3}{1 + \frac{1}{x}} = \frac{3}{1} = 3 \quad \text{onthou } \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$7. \lim_{x \rightarrow \infty} \frac{x^2 - 3x + 1}{3x^2 - x + 2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{3x^2}{x^2} - \frac{x}{x^2} + \frac{2}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{3}{x} + \frac{1}{x^2}}{3 - \frac{1}{x} + \frac{2}{x^2}} = \frac{1}{3} \quad \text{onthou } \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$8. \lim_{x \rightarrow 0} \frac{4}{x} = \infty \quad \text{onthou as jy } \div \text{ met 'n baie klein getal, word die antwoord baie groot}$$

$$9. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \rightarrow 3} (x + 3) = 3 + 3 = 6$$

$$10. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} \\ = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)} \\ = \lim_{x \rightarrow 2} (x^2 + 2x + 4) \\ = (2)^2 + 2(2) + 4 = 12$$

$$11. \lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x+4)(x-1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{(x+4)}{(x+1)} = \frac{1+4}{1+1} = \frac{5}{2}$$

$$12. \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{2x-2} = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{2(x-1)} = \lim_{x \rightarrow 1} \frac{x+2}{2} = \frac{1+2}{2} = \frac{3}{2}$$

$$13. \lim_{x \rightarrow 0} \frac{x^2 - 4}{x - 4} = \frac{0 - 4}{0 - 4} = \frac{-4}{-4} = 1$$

$$14. \lim_{x \rightarrow 2} \frac{x-2}{x^2-4} = \lim_{x \rightarrow 2} \frac{(x-2)}{(x-2)(x+2)} = \lim_{x \rightarrow 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$$

$$15. \lim_{x \rightarrow -1} \frac{2x+1}{x} = \frac{2(-1)+1}{(-1)} = \frac{-1}{-1} = 1$$

$$16. \lim_{x \rightarrow -1} \frac{x^2 - 3x - 4}{x^2 - 1} = \lim_{x \rightarrow -1} \frac{(x+1)(x-4)}{(x-1)(x+1)} = \lim_{x \rightarrow -1} \frac{x-4}{x-1} = \frac{-1-4}{-1-1} = \frac{-5}{-2} = \frac{5}{2}$$

Hoofstuk 1

Differensiaalrekening

Antwoorde 3: Helling van 'n punt op 'n kromme

$$\begin{aligned}
 1.1 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} & f(x) = 4 \quad \text{dus } f(x+h) &= 4 \\
 &= \lim_{h \rightarrow 0} \frac{4-4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{0}{h} \\
 &= \lim_{h \rightarrow 0} 0 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} & f(x) = 3x^2 \quad \text{dus } f(x+h) &= 3(x+h)^2 \\
 &= \lim_{h \rightarrow 0} \frac{3x^2 + 6hx + 3h^2 - 3x^2}{h} & &= 3(x^2 + 2hx + h^2) \\
 &= \lim_{h \rightarrow 0} \frac{3h^2 + 6xh}{h} & &= 3h^2 + 6hx + 3x^2 \\
 &= \lim_{h \rightarrow 0} \frac{h(3h + 6x)}{h} \\
 &= \lim_{h \rightarrow 0} 3h + 6x \\
 &= 6x
 \end{aligned}$$

$$\begin{aligned}
 1.3 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} & f(x) = 3x + 1 \quad \text{dus } f(x+h) &= 3(x+h) + 1 \\
 &= \lim_{h \rightarrow 0} \frac{3h + 3x + 1 - (3x + 1)}{h} & &= 3h + 3x + 1 \\
 &= \lim_{h \rightarrow 0} \frac{3h + 3x + 1 - 3x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3h}{h} \\
 &= \lim_{h \rightarrow 0} 3 \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 1.4 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} & \text{dus } f(x+h) &= 2(x+h)^3 \\
 &= \lim_{h \rightarrow 0} \frac{2h^3 + 6h^2x + 6hx^2 + 2x^3 - 2x^3}{h} & &= 2(x+h)(x+h)^2 \\
 &= \lim_{h \rightarrow 0} \frac{2h^3 + 6h^2x + 6hx^2}{h} & &= 2(x+h)(x^2 + 2hx + h^2) \\
 &= \lim_{h \rightarrow 0} \frac{h(2h^2 + 6hx + 6x^2)}{h} & &= 2(x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3) \\
 &= \lim_{h \rightarrow 0} 2h^2 + 6hx + 6x^2 & &= 2(x^3 + 3hx^2 + 3h^2x + h^3) \\
 &= 6x^2 & &= 2h^3 + 6h^2x + 6hx^2 + 2x^3
 \end{aligned}$$

$$\begin{aligned}
1.5 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{h^2 + 2hx + x^2 + 2h + 2x - 1 - (x^2 + 2x - 1)}{h} \\
&= \lim_{h \rightarrow 0} \frac{h^2 + 2hx + x^2 + 2h + 2x - 1 - x^2 - 2x + 1}{h} \\
&= \lim_{h \rightarrow 0} \frac{2h + 2hx + h^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(h + 2x + 2)}{h} \\
&= \lim_{h \rightarrow 0} h + 2x + 2 \\
&= 2x + 2
\end{aligned}$$

$$f(x) = x^2 + 2x - 1 \text{ dus}$$

$$\begin{aligned}
f(x+h) &= (x+h)^2 + 2(x+h) - 1 \\
&= x^2 + 2hx + h^2 + 2x + 2h - 1
\end{aligned}$$

$$\begin{aligned}
1.6 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{-x^2 - 2hx - h^2 + x + h + 3 - (-x^2 + x + 3)}{h} \\
&= \lim_{h \rightarrow 0} \frac{-x^2 - 2hx - h^2 + x + h + 3 + x^2 - x - 3}{h} \\
&= \lim_{h \rightarrow 0} \frac{h - 2hx - h^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(1 - 2x - h)}{h} \\
&= \lim_{h \rightarrow 0} 1 - 2x - h \\
&= 1 - 2x
\end{aligned}$$

$$f(x) = -x^2 + x + 3 \text{ dus}$$

$$\begin{aligned}
f(x+h) &= -(x+h)^2 + (x+h) + 3 \\
&= -(x^2 + 2hx + h^2) + x + h + 3 \\
&= -x^2 - 2hx - h^2 + x + h + 3
\end{aligned}$$

$$\begin{aligned}
1.7 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{2x - 2x - 2h}{x^2 + xh}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{-2h}{x^2 + xh}}{\frac{h}{1}} \\
&= \lim_{h \rightarrow 0} \frac{-2h}{x^2 + xh} \times \frac{1}{h} \\
&= \lim_{h \rightarrow 0} \frac{-2}{x^2 + hx} \\
&= \frac{-2}{x^2}
\end{aligned}$$

$$f(x) = \frac{2}{x} \text{ dus } f(x+h) = \frac{2}{x+h}$$

$$\begin{aligned}
1.8 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{nx^2 + 2nhx + nh^2 - nx^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{2nhx + nh^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(2nx + nh)}{h} \\
&= \lim_{h \rightarrow 0} 2nx + nh
\end{aligned}$$

$$\begin{aligned}
f(x) &= nx^2 \text{ dus } f(x+h) = n(x+h)^2 \\
&= n(x^2 + 2hx + h^2) \\
&= nx^2 + 2nhx + nh^2
\end{aligned}$$

$$= 2nx$$

$$2.1 f(x) = \frac{1}{2}x^2 \quad \text{by } x = -1$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{2}x^2 + hx + \frac{1}{2}h^2 - (\frac{1}{2}x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{2}x^2 + hx + \frac{1}{2}h^2 - \frac{1}{2}x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{hx + \frac{1}{2}h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(x + \frac{1}{2}h)}{h} \\ &= \lim_{h \rightarrow 0} x + \frac{1}{2}h \\ f'(x) &= x \end{aligned}$$

$$f'(-1) = -1$$

$$2.2 f(x) = -x^2 + 1 \quad \text{by } x = 1$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x^2 - 2hx - h^2 + 1 - (-x^2 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x^2 - 2hx - h^2 + 1 + x^2 - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2hx - h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-2x - h)}{h} \\ &= \lim_{h \rightarrow 0} -2x - h \end{aligned}$$

$$f'(x) = -2x$$

$$f'(1) = -2(1) = -2$$

$$2.3 f(x) = 2(x^2 - 1) \quad \text{by } x = 3$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 4hx + 2h^2 - 2 - (2x^2 - 2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 4hx + 2h^2 - 2 - 2x^2 + 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{4hx + 2h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h} \\ &= \lim_{h \rightarrow 0} 4x + 2h \end{aligned}$$

$$f'(x) = 4x$$

$$f'(3) = 4(3) = 12$$

$$2.4 f(x) = 3x^2 - 4 \quad \text{by } x = 0$$

$$\begin{aligned} f(x) &= \frac{1}{2}x^2 \quad \text{dus } f(x+h) = \frac{1}{2}(x+h)^2 \\ &= \frac{1}{2}(x^2 + 2hx + h^2) \\ &= \frac{1}{2}x^2 + hx + \frac{1}{2}h^2 \end{aligned}$$

$$f(x) = -x^2 + 1$$

$$f(x+h) = -(x+h)^2 + 1$$

$$= -(x^2 + 2hx + h^2) + 1$$

$$= -x^2 - 2hx - h^2 + 1$$

$$f(x) = 2(x^2 - 1) = 2x^2 - 2$$

$$f(x+h) = 2(x+h)^2 - 2$$

$$= 2(x^2 + 2hx + h^2) - 2$$

$$= 2x^2 + 4hx + 2h^2 - 2$$

$$f(x) = 3x^2 - 4$$

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{3x^2 + 6hx + 3h^2 - 4 - (3x^2 - 4)}{h} \\
&= \lim_{h \rightarrow 0} \frac{3x^2 + 6hx + 3h^2 - 4 - 3x^2 + 4}{h} \\
&= \lim_{h \rightarrow 0} \frac{6hx + 3h^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h} \\
&= \lim_{h \rightarrow 0} 6x + 3h
\end{aligned}$$

$$\begin{aligned}
f(x+h) &= 3(x+h)^2 - 4 \\
&= 3(x^2 + 2hx + h^2) - 4 \\
&= 3x^2 + 6hx + 3h^2 - 4
\end{aligned}$$

$$f'(x) = 6x$$

$$f'(0) = 6(0) = 0$$

$$\begin{aligned}
3.1 \quad \frac{f(-1+h) - f(-1)}{h} & \quad f(x) = 2x^3 \text{ dus } f(-1+h) = 2(-1+h)^3 \\
&= \frac{-2 + 6h - 6h^2 + 2h^3 - (-2)}{h} &= 2(-1+h)(-1+h)^2 \\
&= \frac{-2 + 6h - 6h^2 + 2h^3 + 2}{h} &= 2(-1+h)(1 - 2h + h^2) \\
&= \frac{6h - 6h^2 + 2h^3}{h} &= 2(-1 + 2h - h^2 + h - 2h^2 + h^3) \\
&= \frac{h(6 - 6h + 2h^2)}{h} &= 2(-1 + 3h - 3h^2 + h^3) \\
&= 6 - 6h + 2h^2 &= -2 + 6h - 6h^2 + 2h^3 \\
& & f(-1) = 2(-1)^3 = 2(-1) = -2
\end{aligned}$$

$$\begin{aligned}
3.2 \quad \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} \\
&= \lim_{h \rightarrow 0} 6 - 6h + 2h^2 \\
&f'(-1) = 6
\end{aligned}$$

3.3 Die helling van grafiek f by $x = -1$ is 6

4. Gegee: $p'(a)$ se antwoord is $\lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$

$$\text{Onthou: } p'(x) = \lim_{h \rightarrow 0} \frac{p(x+h) - p(x)}{h}$$

$$\begin{aligned}
\therefore p'(a) &= \lim_{h \rightarrow 0} \frac{p(a+h) - p(a)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}
\end{aligned}$$

Met inspeksie volg dan dat $p(a+h) = \sqrt{9+h}$ en dus moet $p(a) = \sqrt{9} = 3$

\therefore was $p(x+h) = \sqrt{x+h}$ en $p(x) = \sqrt{x}$ waar x toe vervang is met 9

Dit volg dus dat $p(x) = \sqrt{x}$ en dat $a = 9$

Hoofstuk 1

Differensiaalrekenne

Antwoorde 4: Differensiasie reëls

1.1 $f(x) = x$

$$f'(x) = 1$$

1.3 $f(x) = 4x^2$

$$f'(x) = 8x$$

1.5 $f(x) = 4$

$$f'(x) = 0$$

1.7 $f(x) = 4 - x$

$$f'(x) = -1$$

1.9 $f(x) = 4 - \frac{1}{x}$

$$f(x) = 4 - 1 \cdot x^{-1}$$

$$f'(x) = x^{-2} = \frac{1}{x^2}$$

2.1 $y = x^{-2}$

$$\frac{dy}{dx} = -2x^{-3} = \frac{-2}{x^3}$$

2.3 $y = x^{0,3}$

$$\frac{dy}{dx} = 0,3x^{0,3-1} = 0,3x^{-0,7} = \frac{0,3}{x^{0,7}}$$

2.5 $y = 3x^{-3}$

$$\frac{dy}{dx} = 3 \cdot (-3)x^{-4} = \frac{-9}{x^4}$$

2.7 $y = \frac{1}{2x^2}$

$$y = \frac{1 \cdot x^{-2}}{2}$$

$$\frac{dy}{dx} = \frac{-2x^{-3}}{2} = \frac{-1}{x^3}$$

2.9 $y = \sqrt[3]{8x}$

$$y = 2x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{2}{1} \cdot \frac{1}{3} x^{\frac{1}{3}-1}$$

$$= \frac{2}{3} x^{-\frac{2}{3}} = \frac{2}{3x^{\frac{2}{3}}} = \frac{2}{3\sqrt[3]{x^2}}$$

3.1 $\frac{d}{dx}(2x - 3)$

$$= 2$$

3.3 $\frac{d}{dx}[x^4 - 3x^2]$

$$= 4x^3 - 6x$$

3.5 $\frac{d}{dx}[ax^3 + bx^2 + cx + d]$

$$= 3ax^2 + 2bx + c$$

1.2 $f(x) = 4x$

$$f'(x) = 4$$

1.4 $f(x) = 4x^3$

$$f'(x) = 12x^2$$

1.6 $f(x) = -4x$

$$f'(x) = -4$$

1.8 $f(x) = 4x^{-1}$

$$f'(x) = -4x^{-2} = \frac{-4}{x^2}$$

1.10 $f(x) = 4x - \frac{4}{x}$

$$f(x) = 4x - 4 \cdot x^{-1}$$

$$f'(x) = 4 + 4x^{-2} = 4 + \frac{4}{x^2}$$

2.2 $y = x^{100}$

$$\frac{dy}{dx} = 100x^{99}$$

2.4 $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3} x^{\frac{1}{3}-1} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

2.6 $y = 3x - x^3$

$$\frac{dy}{dx} = 3 - 3x^2$$

2.8 $x^2y = 2$

$$y = \frac{2}{x^2} = 2 \cdot x^{-2}$$

$$\frac{dy}{dx} = -4x^{-3} = \frac{-4}{x^3}$$

2.10 $y = 2\sqrt{x}$

$$y = 2x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{2}{1} \cdot \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

3.2 $\frac{d}{dx}(4x + 5)$

$$= 4$$

3.4 $\frac{d}{dx}[mx + c]$

$$= m$$

3.6 $\frac{d}{dx}[(x+2)(x-4)]$

$$= \frac{d}{dx}[x^2 - 2x - 8]$$

$$\begin{aligned}
3.7 \quad & \frac{d}{dx} [(x^2 - 3)(x + 1)] \\
&= \frac{d}{dx} [x^3 + x^2 - 3x - 3] \\
&= 3x^2 + 2x - 3
\end{aligned}$$

$$\begin{aligned}
3.9 \quad & \frac{d}{dx} \left[\frac{4x^2 - 1}{2x + 1} \right] \\
&= \frac{d}{dx} \left[\frac{(2x - 1)(2x + 1)}{(2x + 1)} \right] \\
&= \frac{d}{dx} [2x - 1] \\
&= 2
\end{aligned}$$

$$\begin{aligned}
4.1 \quad & D_x [x^2 - 3x + 2] \\
&= 2x - 3
\end{aligned}$$

$$\begin{aligned}
4.2 \quad & D_x [8x^3 + 3x^2 - 11] \\
&= 24x^2 + 6x
\end{aligned}$$

$$\begin{aligned}
4.4 \quad & D_x \left[\sqrt{x^2 - 6x + 9} \right] \\
&= D_x \left[\sqrt{(x - 3)(x - 3)} \right] \\
&= D_x \left[\sqrt{(x - 3)^2} \right] \\
&= D_x [x - 3] \\
&= 1
\end{aligned}$$

$$\begin{aligned}
4.5 \quad & D_x \left[\frac{x^2 - 3x - 4}{x + 1} \right] \\
&= D_x \left[\frac{(x + 1)(x - 4)}{(x + 1)} \right] \\
&= D_x [x - 4] \\
&= 1
\end{aligned}$$

$$\begin{aligned}
4.7 \quad & D_x [5\sqrt{x} - \sqrt{5x}] \\
&= D_x \left[5x^{\frac{1}{2}} - \sqrt{5}x^{\frac{1}{2}} \right] \\
&= \frac{5}{1} \cdot \frac{1}{2}x^{-\frac{1}{2}} - \frac{\sqrt{5}}{1} \cdot \frac{1}{2}x^{-\frac{1}{2}} \\
&= \frac{5}{2x^{\frac{1}{2}}} - \frac{\sqrt{5}}{2x^{\frac{1}{2}}} = \frac{5 - \sqrt{5}}{2\sqrt{x}}
\end{aligned}$$

$$\begin{aligned}
4.9 \quad & D_x \left[\frac{x^2 - 3x - 4}{x} \right] \\
&= D_x \left[\frac{x^2}{x} - \frac{3x}{x} - \frac{4}{x} \right] \\
&= D_x \left[x - 3 - \frac{4}{x} \right] \\
&= D_x [x - 3 - 4x^{-1}] \\
&= 1 + 4x^{-2} = 1 + \frac{4}{x^2}
\end{aligned}$$

$$= 2x - 2$$

$$\begin{aligned}
3.5 \quad & \frac{d}{dx} [(2x - 1)^2] \\
&= \frac{d}{dx} [4x^2 - 4x + 1] \\
&= 8x - 4
\end{aligned}$$

$$\begin{aligned}
3.10 \quad & \frac{d}{dx} \left[\sqrt{x} - \frac{1}{\sqrt{x}} \right] \\
&= \frac{d}{dx} \left[x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}} \right] \\
&= \frac{d}{dx} \left[x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right] \\
&= \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} \\
&= \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{2x^{\frac{3}{2}}} \\
&= \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x^3}}
\end{aligned}$$

$$\begin{aligned}
4.3 \quad & D_x [3x^3 - 4x^2 + 2x + 1] \\
&= 9x^2 - 8x + 2
\end{aligned}$$

$$\begin{aligned}
4.6 \quad & D_x \left[\frac{1}{x^2} - \frac{1}{x} \right] \\
&= D_x [1 \cdot x^{-2} - 1 \cdot x^{-1}] \\
&= -2x^{-3} + x^{-2} \\
&= \frac{-2}{x^3} + \frac{1}{x^2}
\end{aligned}$$

$$\begin{aligned}
4.8 \quad & D_x \left[\frac{3x^3 - 4x^4}{x^4} \right] \\
&= D_x \left[\frac{3x^3}{x^4} - \frac{4x^4}{x^4} \right] \\
&= D_x \left[\frac{3}{x} - 4 \right] \\
&= D_x [3 \cdot x^{-1} - 4] \\
&= -3x^{-2} = \frac{-3}{x^2}
\end{aligned}$$

$$\begin{aligned}
4.10 \quad & D_x \left[\sqrt{x^3} - \sqrt[3]{x^2} \right] \\
&= D_x \left[x^{\frac{3}{2}} - x^{\frac{2}{3}} \right] \\
&= \frac{3}{2}x^{\frac{3}{2}-1} - \frac{2}{3}x^{\frac{2}{3}-1} \\
&= \frac{3}{2}x^{\frac{1}{2}} - \frac{2}{3}x^{-\frac{1}{3}} \\
&= \frac{3}{2}x^{\frac{1}{2}} - \frac{2}{3x^{\frac{1}{3}}} \\
&= \frac{3}{2}\sqrt{x} - \frac{2}{3\sqrt[3]{x}}
\end{aligned}$$

$$5. \quad xy + y = x^2 - 1$$

$$y(x+1) = x^2 - 1$$

$$\frac{y(x+1)}{(x+1)} = \frac{x^2-1}{(x+1)}$$

$$y = \frac{(x-1)(x+1)}{(x+1)}$$

$$y = x - 1$$

$$\frac{dy}{dx} = 1$$

$$7. \quad f(y) = \frac{16}{y} = 16y^{-1}$$

$$f'(y) = -16y^{-2}$$

$$= \frac{-16}{y^2}$$

$$4f'(y) = \frac{4}{1} \left(\frac{-16}{y^2} \right) = \frac{-64}{y^2}$$

$$f'(4y) = \frac{-16}{(4y)^2}$$

$$= \frac{-16}{16y^2}$$

$$= \frac{-1}{y^2}$$

$$\therefore f'(4y) - 4f'(y) = \frac{-1}{y^2} - \frac{-64}{y^2}$$

$$= \frac{-1+64}{y^2} = \frac{63}{y^2}$$

$$9. \quad \sqrt{x} = y^{\frac{1}{3}} \quad \text{en} \quad y = z^{-3}$$

$$9.1 \quad \sqrt{x} = \sqrt[3]{y}$$

$$\sqrt{x}^3 = (\sqrt[3]{y})^3$$

$$x^{\frac{3}{2}} = y$$

$$y = x^{\frac{3}{2}}$$

$$\therefore \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{3}{2}\sqrt{x}$$

$$9.3 \quad y = z^{-3}$$

$$\frac{dy}{dz} = -3z^{-4}$$

$$= \frac{-3}{z^4}$$

$$6. \quad 4t^2 - 2tr + r - 1 = 0$$

$$-2tr + r = 1 - 4t^2$$

$$r(-2t+1) = 1 - 4t^2$$

$$\frac{r(-2t+1)}{(-2t+1)} = \frac{1-4t^2}{(-2t+1)}$$

$$r = \frac{(1-2t)(1+2t)}{(-2t+1)}$$

$$r = 1 + 2t$$

$$\frac{dr}{dt} = 2$$

$$8. \quad y = 2x(3-x) \quad \text{en} \quad z = \frac{y}{2}$$

$$y = 6x - 2x^2$$

$$8.1 \quad \frac{dy}{dx} = 6 - 4x$$

$$8.2 \quad \frac{dz}{dy} = \frac{1}{2}$$

$$8.3 \quad y = 6x - 2x^2 \dots (1) \quad \text{en} \quad \frac{z}{1} = \frac{y}{2}$$

Stel y in (1)

$$2z = y \dots (2)$$

$$2z = 6x - 2x^2$$

$$\frac{2z}{2} = \frac{6x}{2} - \frac{2x^2}{2}$$

$$z = 3x - x^2$$

$$\frac{dz}{dx} = 3 - 2x$$

$$9.2 \quad y = x^{\frac{3}{2}} \dots (1) \quad \text{en} \quad y = z^{-3} \dots (2)$$

stel y in (1)

$$z^{-3} = x^{\frac{3}{2}}$$

$$z^{-3} = \sqrt{x^3}$$

$$(z^{-3})^2 = (\sqrt{x^3})^2$$

$$z^{-6} = x^3$$

$$\sqrt[3]{z^{-6}} = \sqrt[3]{x^3}$$

$$z^{-2} = x$$

$$x = z^{-2}$$

$$\frac{dx}{dz} = -2z^{-3}$$

$$= \frac{-2}{z^3}$$

Hoofstuk 1

Differensiaalrekening

Antwoorde 5a: Vergelyking van 'n raaklyn aan 'n kromme

1. Bereken die vergelyking van die raaklyn in elk van die volgende gevalle:

1.1 $f(x) = x^2 - 2x + 1$ by $x = -1$

RP $(-1; y)$

$$f(-1) = (-1)^2 - 2(-1) + 1 = 1 + 2 + 1 = 4$$

RP $(-1; 4)$

$$f'(x) = 2x - 2$$

$$f'(-1) = 2(-1) - 2 = -2 - 2 = -4$$

$$m_{\text{raaklyn}} = f'(-1) = -4 \quad \text{en} \quad \text{RP}(-1; 4)$$

$$\text{Raaklyn: } y - y_1 = m(x - x_1)$$

$$y - 4 = -4(x + 1)$$

$$y - 4 = -4x - 4$$

$$y = -4x$$

1.2 $f(x) = x^3 - 2x^2 - 9x + 18$ by $x = 1$

RP $(1; y)$

$$f(1) = (1)^3 - 2(1)^2 - 9(1) + 18 = 1 - 2 - 9 + 18 = 8$$

RP $(1; 8)$

$$f'(x) = 3x^2 - 4x - 9$$

$$f'(1) = 3(1)^2 - 4(1) - 9 = 3 - 4 - 9 = -10$$

$$m_{\text{raaklyn}} = f'(1) = -10 \quad \text{en} \quad \text{RP}(1; 8)$$

$$\text{Raaklyn: } y - y_1 = m(x - x_1)$$

$$y - 8 = -10(x - 1)$$

$$y - 8 = -10x + 10$$

$$y = -10x + 18$$

1.3 $g(x) = 4\sqrt{x}$ by $x = 4$

RP $(4; y)$

$$g(4) = 4\sqrt{4} = 4(2) = 8$$

RP $(4; 8)$

$$g(x) = 4\sqrt{x} = 4x^{\frac{1}{2}}$$

$$g'(x) = \frac{4}{1} \left(\frac{1}{2}\right) x^{\frac{1}{2}-1} = 2x^{-\frac{1}{2}} = \frac{2}{x^{\frac{1}{2}}} = \frac{2}{\sqrt{x}}$$

$$g'(4) = \frac{2}{\sqrt{4}} = \frac{2}{2} = 1$$

$$m_{\text{raaklyn}} = g'(4) = 1 \quad \text{en} \quad \text{RP}(4; 8)$$

Raaklyn: $y - y_1 = m(x - x_1)$

$$y - 8 = 1(x - 4)$$

$$y - 8 = x - 4$$

$$y = x + 4$$

1.4 $p(x) = \frac{3}{x}$ by $x = 1$

RP (1;y)

$$p(1) = \frac{3}{1} = 3$$

RP (1;3)

$$p(x) = \frac{3}{x} = 3x^{-1}$$

$$p'(x) = -3x^{-2} = -\frac{3}{x^2}$$

$$p'(1) = -\frac{3}{(1)^2} = -\frac{3}{1} = -3$$

$m_{raaklyn} = p'(1) = -3$ en RP(1;3)

Raaklyn: $y - y_1 = m(x - x_1)$

$$y - 3 = -3(x - 1)$$

$$y - 3 = -3x + 3$$

$$y = -3x + 6$$

2.1 $f(x) = 2x^2 - 3x$ en $m_{raaklyn} = f'(x_{RP}) = 5$

$$f'(x) = 4x - 3$$

$$f(2) = 2(2)^2 - 3(2)$$

$$5 = 4x - 3$$

$$= 8 - 6$$

$$8 = 4x$$

$$= 2$$

$$2 = x$$

punt is (2; 2)

2.2 $f(x) = 2x^2 - 3x$ en $m_{raaklyn} = f'(x) = -1$

$$f'(x) = 4x - 3$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right)$$

$$-1 = 4x - 3$$

$$= \frac{2}{1}\left(\frac{1}{4}\right) - \frac{3}{1}\left(\frac{1}{2}\right)$$

$$2 = 4x$$

$$= \frac{1}{2} - \frac{3}{2} = -1$$

$$\frac{1}{2} = x$$

punt is $\left(\frac{1}{2}; -1\right)$

3.

Stel AB: $y = -2x + 1$ en $f(x) = x^2 + 2x - 3$

$$m_{AB} = -2$$

$$m_{raaklyn} = m_{AB} = -2$$

raaklyn // AB

$$m_{raaklyn} = f'(x) = -2$$

$$f'(x) = 2x + 2$$

$$f(-2) = (-2)^2 + 2(-2) - 3$$

$$-2 = 2x + 2$$

$$= 4 - 4 - 3$$

$$-4 = 2x$$

$$= -3$$

$$-2 = x$$

punt is (-2; -3)

4.

Stel AB: $y = x$ en $f(x) = 4 - x^2$

$$m_{AB} = 1$$

$$m_{\text{raaklyn}} = -1$$

raaklyn \perp AB

$$m_{\text{raaklyn}} = f'(x) = -1$$

$$f'(x) = -2x$$

$$f\left(\frac{1}{2}\right) = 4 - \left(\frac{1}{2}\right)^2$$

$$-1 = -2x$$

$$= 4 - \frac{1}{4}$$

$$\frac{1}{2} = x$$

$$= 3\frac{3}{4}$$

punt is $\left(\frac{1}{2}; 3\frac{3}{4}\right)$

5. $m_{\text{raaklyn}} = \tan 45^\circ$ Die raaklyn se inklinasiehoek is 45°

$$\therefore m_{\text{raaklyn}} = 1$$

$$f(x) = 12x^3$$

$$\therefore f'(x) = 36x^2$$

formule om helling by enige x -waarde te bereken

maar $f'(x) = m_{\text{raaklyn}} = 1$ helling van punt is dieselfde as helling van die raaklyn

$$1 = 36x^2$$

$$\frac{1}{36} = x^2$$

$$\pm \sqrt{\frac{1}{36}} = \sqrt{x^2}$$

$$\pm \frac{1}{6} = x$$

Hoofstuk 1

Differensiaalrekenen

Antwoorde 5b: Gemeng deel 1

$$\begin{aligned}
 1.1 \quad \lim_{h \rightarrow 0} \frac{4(1+h)^2 - 4}{h} &= \lim_{h \rightarrow 0} \frac{4(1+2h+h^2) - 4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4+8h+4h^2 - 4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{8h+4h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(8+4h)}{h} \\
 &= \lim_{h \rightarrow 0} 8 + 4h \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h} &= \lim_{h \rightarrow 0} \frac{3(x^2+2hx+h^2) - 3x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3x^2+6hx+3h^2 - 3x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{6hx+3h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(6x+3h)}{h} \\
 &= \lim_{h \rightarrow 0} 6x + 3h \\
 &= 6x
 \end{aligned}$$

$$1.3 \quad \lim_{x \rightarrow -\frac{1}{2}} \frac{2x^2 - x - 1}{2x + 1} = \lim_{x \rightarrow -\frac{1}{2}} \frac{(2x+1)(x-1)}{(2x+1)} = \lim_{x \rightarrow -\frac{1}{2}} x - 1 = -\frac{1}{2} - 1 = -1\frac{1}{2}$$

$$g(x) = \frac{-5}{x}$$

$$\begin{aligned}
 2.1 \quad g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-5}{x+h} - \left(\frac{-5}{x}\right)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-5}{x+h} + \frac{5}{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-5x+5(x+h)}{x(x+h)}}{\frac{h}{1}} \\
 &= \lim_{h \rightarrow 0} \frac{-5x+5x+5h}{x^2+hx} \times \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5h}{x^2+hx} \times \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5}{x^2+hx} \\
 &= \frac{5}{x^2}
 \end{aligned}$$

$$g(x+h) = \frac{-5}{x+h}$$

$$2.2 \quad g(x) = \frac{-5}{x}$$

$$g(-1) = \frac{-5}{-1} = 5$$

$$RP(-1; 5)$$

$$g'(x) = \frac{5}{x^2}$$

$$g'(-1) = \frac{5}{(-1)^2} = \frac{5}{1} = 5$$

$$m_{raaklyn} = 5 \text{ en } (-1; 5)$$

Raaklyn:

$$y - 5 = 5(x + 1)$$

$$y - 5 = 5x + 5$$

$$y = 5x + 10$$

$$y - y_1 = m(x - x_1)$$

$$\begin{aligned}
 3.1 \quad \frac{d}{dx} (3x^2 - 7x - 6)(x - 3)^{-1} &= \frac{d}{dx} \frac{(3x^2 - 7x - 6)}{(x - 3)^1} \\
 &= \frac{d}{dx} \frac{(3x+2)(x-3)}{(x-3)} \\
 &= \frac{d}{dx} (3x + 2) \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
3.2 \quad D_x(6\sqrt[3]{x^2}) &= D_x(6x^{\frac{2}{3}}) \\
&= \frac{6}{1} \cdot \frac{2}{3} x^{\frac{2}{3}-1} \\
&= 4x^{-\frac{1}{3}} \\
&= \frac{4}{x^{\frac{1}{3}}} \\
&= \frac{4}{\sqrt[3]{x}}
\end{aligned}$$

$$\begin{aligned}
3.4 \quad y &= \frac{x^3-x^2-2x}{x+1} \\
&= \frac{x(x^2-x-2)}{x+1} \\
&= \frac{x(x+1)(x-2)}{(x+1)} \\
&= x(x-2) \\
&= x^2-2x \\
\frac{dy}{dx} &= 2x-2
\end{aligned}$$

$$\begin{aligned}
3.6 \quad g(x) &= \frac{1-2x+\sqrt{x}}{x^2} \\
&= \frac{1}{x^2} - \frac{2x}{x^2} + \frac{x^{\frac{1}{2}}}{x^2} \\
&= \frac{1}{x^2} - \frac{2}{x} + x^{\frac{1}{2}} \cdot x^{-2} \\
&= x^{-2} - 2x^{-1} + x^{-\frac{3}{2}} \\
g'(x) &= -2x^{-3} + 2x^{-2} - \frac{3}{2}x^{-\frac{5}{2}} \\
&= -\frac{2}{x^3} + \frac{2}{x^2} - \frac{3}{2x^{\frac{5}{2}}} \\
&= -\frac{2}{x^3} + \frac{2}{x^2} - \frac{3}{2\sqrt{x^5}}
\end{aligned}$$

$$\begin{aligned}
3.8 \quad xy &= 5 \\
y &= \frac{5}{x} \\
&= 5x^{-1} \\
\frac{dy}{dx} &= -5x^{-2} \\
&= -\frac{5}{x^2}
\end{aligned}$$

$$\begin{aligned}
4.2 \quad f(x) &= x^2 - 4x \\
f'(x) &= 2x - 4 \\
f'(-1) &= 2(-1) - 4 = -6
\end{aligned}$$

$$\begin{aligned}
3.3 \quad h(x) &= (x^3 - \frac{3}{x})(x^3 + \frac{3}{x}) \\
&= x^6 - \frac{9}{x^2} \\
&= x^6 - 9x^{-2} \\
h'(x) &= 6x^5 + 18x^{-3} \\
&= 6x^5 + \frac{18}{x^3}
\end{aligned}$$

$$\begin{aligned}
3.5 \quad D_x\left(\sqrt{x} + \frac{6}{\sqrt[3]{x^2}} + \pi x^3\right) &= D_x\left(x^{\frac{1}{2}} + \frac{6}{x^{\frac{2}{3}}} + \pi x^3\right) \\
&= D_x\left(x^{\frac{1}{2}} + 6x^{-\frac{2}{3}} + \pi x^3\right)
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2}x^{-\frac{1}{2}} + \frac{6}{1} \cdot \left(-\frac{2}{3}\right)x^{-\frac{5}{3}} + 3\pi x^2 \\
&= \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-\frac{5}{3}} + 3\pi x^2 \\
&= \frac{1}{2x^{\frac{1}{2}}} - \frac{4}{x^{\frac{5}{3}}} + 3\pi x^2 \\
&= \frac{1}{2\sqrt{x}} - \frac{4}{\sqrt[3]{x^5}} + 3\pi x^2
\end{aligned}$$

$$\begin{aligned}
3.7 \quad D_x\left(\frac{(2x^2-5)(3x+2)}{x^2}\right) &= D_x\left(\frac{6x^3+4x^2-15x-10}{x^2}\right) \\
&= D_x\left(\frac{6x^3}{x^2} + \frac{4x^2}{x^2} - \frac{15x}{x^2} - \frac{10}{x^2}\right) \\
&= D_x\left(6x + 4 - \frac{15}{x} - \frac{10}{x^2}\right) \\
&= D_x(6x + 4 - 15x^{-1} - 10x^{-2}) \\
&= 6 + 15x^{-2} + 20x^{-3} \\
&= 6 + \frac{15}{x^2} + \frac{20}{x^3}
\end{aligned}$$

$$\begin{aligned}
4.1 \quad f(x) &= x^2 - 4x \\
f(-1) &= (-1)^2 - 4(-1) = 5 \\
f(1) &= (1)^2 - 4(1) = -3 \\
\text{gemiddelde helling} &= \frac{y_2-y_1}{x_2-x_1} \\
&= \frac{-3-5}{1+1} \\
&= \frac{-8}{2} = -4
\end{aligned}$$

5. $y = -\frac{1}{2}x + 4$

$$m = -\frac{1}{2}$$

$m_{\text{raaklyn}} = 2$ lyne \perp mekaar

$$f'(x) = 2$$

$$y = f(x) = x^2 - 4x$$

$$f'(x) = 2x - 4$$

$$f'(x) = m_{\text{raaklyn}} = 2$$

$$2 = 2x - 4$$

$$6 = 2x$$

$$3 = x$$

die raaklyn raak f by $x = 3$

$$f(x) = x^2 - 4x$$

$$f(3) = (3)^2 - 4(3) = -3$$

$RP(3; -3)$ en $m_{\text{raaklyn}} = 2$

Raaklyn: $y - y_1 = m(x - x_1)$

$$y + 3 = 2(x - 3)$$

$$y + 3 = 2x - 6$$

$$y = 2x - 9$$

Meer oor “Wiskunde Anibrand Antwoordboek Graad 12” en die outeur.

Ek is reeds vir 28 jaar betrokke by Wiskunde-onderrig vir graad 8 tot graad 12 leerders. Die afgelope 10 jaar is ek verbonde aan Hoërskool Die Wilgers in Pretoria, waar ek ‘n Wiskunde Akademie bedryf met een groep in elke graad.

Met die aanvang van die nuwe KABV sillabus in 2007 het ek begin om al my notas vir my Wiskunde-onderrig elektronies saam te stel met behulp van innoverende sagteware sodat dit alle onderwerpe met grafika en voorbeelde volledig verduidelik. Die graad 12 Wiskunde Anibrand Antwoordboek bied volledig uitgewerkte, verduidelikende antwoorde vir al die huiswerk probleme in die graad 12 “Wiskunde Anibrand Notaboek”.

Leerders wat hierdie boek gebruik om hulle huiswerk probleme te merk, kan uit die antwoorde self sien waar hulle gefouteer het en dan hulle foute korrigeer.

Ek gebruik hierdie antwoorde die afgelope 5 jaar in my klasaanbieding vir die graad 12 leerders. Dit stel my in staat om die antwoorde konstant te verbeter, soos wat ek dit in die klassituasie as nodig ervaar.

Die volledige antwoorde op huiswerkprobleme help leerders om selfvertroue in die vak te kry want hulle kan hulle probleme identifiseer wanneer hulle huiswerk doen, dit uitsorteer en dan voortgaan met die res van die huiswerk probleme.

Hierdie boek, saam met die Notaboek, is die antwoord vir alle graad 12 leerders wat wil presteer in Wiskunde en ook vir alle Wiskunde onderwysers wat sonder moeite ‘n kwaliteit Wiskunde klasaanbieding vir leerders wil bied.

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