

Wiskunde Anibrand

Antwoordboek Graad 11



Annie Bothma

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Annie Bothma

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Hoofstuk 1

Eksponentes en Wortel

Antwoorde 1: Wortels

1. $(2\sqrt{3} + 3)(2\sqrt{3} - 2)$ 2 hakies met 2 terme EMUL/EBBA

$$= 2\sqrt{3}(2\sqrt{3}) + 3 \cdot 2\sqrt{3} - 2 \cdot 2\sqrt{3} - 2 \cdot 3$$

$$= 12 + 6\sqrt{3} - 4\sqrt{3} - 6 \quad \text{Tel gelyksoortiges op}$$

$$= 6 + 2\sqrt{3}$$

2. $\frac{\sqrt{18}-\sqrt{98}}{\sqrt{8}} - \sqrt{2\frac{1}{2} + \frac{9}{16}}$

$$= \frac{\sqrt{2 \times 9} - \sqrt{2 \times 49}}{\sqrt{2 \times 4}} - \sqrt{\frac{5}{2} + \frac{9}{16}}$$

$$= \frac{\sqrt{2} \times \sqrt{9} - \sqrt{2} \times \sqrt{49}}{\sqrt{2} \times \sqrt{4}} - \sqrt{\frac{40}{16} + \frac{9}{16}}$$

$$= \frac{3\sqrt{2} - 7\sqrt{2}}{2\sqrt{2}} - \sqrt{\frac{49}{16}}$$

$$= \frac{-4\sqrt{2}}{2\sqrt{2}} - \frac{7}{4}$$

$$= -\frac{2}{1} - \frac{7}{4} = -\frac{15}{4}$$

4. $\frac{\sqrt{12}-\sqrt{3}}{\sqrt{27}}$

$$= \frac{\sqrt{4 \times 3} - \sqrt{3}}{\sqrt{9 \times 3}}$$

$$= \frac{\sqrt{4} \times \sqrt{3} - \sqrt{3}}{\sqrt{9} \times \sqrt{3}}$$

$$= \frac{2\sqrt{3} - \sqrt{3}}{3\sqrt{3}}$$

$$= \frac{1\sqrt{3}}{3\sqrt{3}}$$

$$= \frac{1}{3}$$

6. $(\sqrt{12} - \sqrt{3})^2$

$$= (\sqrt{4 \times 3} - \sqrt{3})^2 \quad \text{vereenvoudig eers wortels}$$

$$= (2\sqrt{3} - \sqrt{3})^2 \quad \text{tel op}$$

$$= (\sqrt{3})^2$$

$$= 3$$

3. $\sqrt{108} - \sqrt{625} + \sqrt{432}$

$$= \sqrt{36 \times 3} - \sqrt{5^4} + \sqrt{144 \times 3}$$

$$= \sqrt{36} \times \sqrt{3} - 5^2 + \sqrt{144} \times \sqrt{3}$$

$$= 6\sqrt{3} - 5^2 + 12\sqrt{3}$$

$$= 18\sqrt{3} - 25$$

5. $\frac{\sqrt{48}-\sqrt{32}}{\sqrt{12}-\sqrt{8}}$

$$= \frac{\sqrt{16 \times 3} - \sqrt{16 \times 2}}{\sqrt{4 \times 3} - \sqrt{4 \times 2}}$$

$$= \frac{\sqrt{16} \times \sqrt{3} - \sqrt{16} \times \sqrt{2}}{\sqrt{4} \times \sqrt{3} - \sqrt{4} \times \sqrt{2}}$$

$$= \frac{4\sqrt{3} - 4\sqrt{2}}{2\sqrt{3} - 2\sqrt{2}}$$

$$= \frac{4(\sqrt{3} - \sqrt{2})}{2(\sqrt{3} - \sqrt{2})}$$

$$= \frac{4}{2} = 2$$

7. $\sqrt{3}(\sqrt{48} + 3\sqrt{75} - 2\sqrt{108})$

$$= \sqrt{3}(\sqrt{16 \times 3} + 3\sqrt{25 \times 3} - 2\sqrt{36 \times 3})$$

$$= \sqrt{3}(4\sqrt{3} + 3 \cdot 5\sqrt{3} - 2 \cdot 6\sqrt{3})$$

$$= \sqrt{3}(4\sqrt{3} + 15\sqrt{3} - 12\sqrt{3})$$

$$= \sqrt{3}(7\sqrt{3})$$

$$= 7 \cdot 3$$

$$= 21$$

8. $\frac{(\sqrt{2}+2)^2}{2} - \sqrt{8}$ werk ()² uit - 3 terme

$$= \frac{2+4\sqrt{2}+4}{2} - \sqrt{4 \times 2}$$

$$= \frac{6+4\sqrt{2}}{2} - 2\sqrt{2} \quad \text{faktoriseer}$$

$$= \frac{2(3+2\sqrt{2})}{2} - 2\sqrt{2} \quad \text{kanselleer}$$

$$= 3 + 2\sqrt{2} - 2\sqrt{2} \quad \text{tel op}$$

$$= 3$$

9. $(1 + \sqrt{3} - \sqrt{27})(1 + 2\sqrt{3})$

$$= (1 + \sqrt{3} - \sqrt{9 \times 3})(1 + 2\sqrt{3})$$

$$= (1 + \sqrt{3} - 3\sqrt{3})(1 + 2\sqrt{3})$$

$$= (1 - 2\sqrt{3})(1 + 2\sqrt{3}) \quad \text{EBBA}$$

$$= -11 \quad \text{se kortpad}$$

10. $\sqrt[3]{64x^{600}} + \sqrt[3]{-512}$ (-)^{ewe} = +

$$= \sqrt[3]{2^6 x^{600}} + \sqrt[3]{(-2)^9} \quad \text{(-)^{onewe} = -}$$

$$= 2^{\frac{6}{3}} x^{\frac{600}{3}} + (-2)^{\frac{9}{3}}$$

$$= 2x^{100} + (-2)^1$$

$$= 2x^{100} - 2$$

11. $\sqrt[3]{a^{xy}} + \sqrt[3]{a^{2y}}$

$$= a^{\frac{xy}{3}} + a^{\frac{2y}{3}}$$

$$= a^y + a^y \quad \text{Tel gelyksoortige terme op}$$

$$= 2a^y \quad \rightarrow x + x = 2x \text{ en ook}$$

$$a^y + a^y = 2a^y$$

12. $(\sqrt{(\sqrt{13} - \sqrt{5})} \cdot \sqrt{(\sqrt{13} + \sqrt{5})})^{-2}$ wortelwet

$$= (\sqrt{(\sqrt{13} - \sqrt{5}) \cdot (\sqrt{13} + \sqrt{5})})^{-2} \quad \text{EBBA kortpad}$$

$$= (\sqrt{(13 - 5)})^{-2}$$

$$= (\sqrt{8})^{-2}$$

$$= \frac{1}{(\sqrt{8})^2}$$

$$= \frac{1}{8}$$

13. $\frac{\sqrt{8a^3} - \sqrt{3a^2b}}{\sqrt{12b^3} - \sqrt{32ab^2}}$

$$= \frac{\sqrt{4 \times 2 \times a^2 \times a} - \sqrt{3 \times a^2 \times b}}{\sqrt{4 \times 3 \times b^2 \times b} - \sqrt{16 \times 2 \times a \times b^2}}$$

$$= \frac{2a\sqrt{2a} - a\sqrt{3b}}{2b\sqrt{3b} - 4b\sqrt{2a}}$$

$$= \frac{a(2\sqrt{2a} - \sqrt{3b})}{2b(\sqrt{3b} - 2\sqrt{2a})}$$

$$= \frac{a(2\sqrt{2a} - \sqrt{3b})}{-2b(2\sqrt{2a} - \sqrt{3b})}$$

$$= \frac{a}{-2b}$$

14.

$\sqrt[3]{4}$	$\sqrt[4]{5}$	
$4^{\frac{1}{3}}$	$5^{\frac{1}{4}}$	magsvorm
$4^{\frac{4}{12}}$	$5^{\frac{3}{12}}$	nomers van breuke dieselfde
$\sqrt[12]{4^4}$	$\sqrt[12]{5^3}$	terug na wortelvorm
$\sqrt[12]{256}$	$\sqrt[12]{125}$	vereenvoudig onder wortels

$$\sqrt[3]{4} > \sqrt[4]{5}$$

15. $\frac{a}{\sqrt{2}} + 4 = \frac{17\sqrt{2}}{5-\sqrt{8}}$

$$\frac{a}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + 4 = \frac{17\sqrt{2}}{5-2\sqrt{2}} \quad \text{rasionaliseer}$$

$$\frac{\sqrt{2}a}{2} + 4 = \frac{17\sqrt{2}}{(5-2\sqrt{2})} \times \frac{(5+2\sqrt{2})}{(5+2\sqrt{2})} \quad \text{rasionaliseer}$$

$$\frac{\sqrt{2}a}{2} + 4 = \frac{17\sqrt{2}(5+2\sqrt{2})}{(5-2\sqrt{2})(5+2\sqrt{2})}$$

$$\frac{\sqrt{2}a}{2} + 4 = \frac{85\sqrt{2}+34.2}{25-4.2}$$

16. $a = \frac{3-\sqrt{7}}{\sqrt{2}}$ en $b = \frac{3+\sqrt{7}}{\sqrt{2}}$

$$a \cdot b = \frac{3-\sqrt{7}}{\sqrt{2}} \times \frac{3+\sqrt{7}}{\sqrt{2}}$$

$$= \frac{(3-\sqrt{7}) \cdot (3+\sqrt{7})}{2}$$

$$= \frac{9-7}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

$$\frac{\sqrt{2}a}{2} + 4 = \frac{85\sqrt{2}+68}{17}$$

$$\frac{\sqrt{2}a}{2} + 4 = \frac{17(5\sqrt{2}+4)}{17}$$

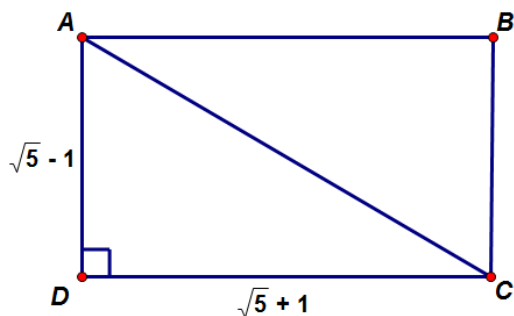
$$\frac{\sqrt{2}a}{2} + 4 = 5\sqrt{2} + 4$$

$$\frac{\sqrt{2}a}{2} = \frac{5\sqrt{2}}{1}$$

$$\sqrt{2}a = 10\sqrt{2}$$

$$a = 10$$

17.



$$AC^2 = (\sqrt{5} - 1)^2 + (\sqrt{5} + 1)^2 \quad \text{pyth}$$

$$AC^2 = 5 - 2\sqrt{5} + 1 + 5 + 2\sqrt{5} + 1$$

$$AC^2 = 12$$

$$\sqrt{AC^2} = \sqrt{12}$$

$$AC = 2\sqrt{3}$$

$$18.1 \quad \frac{5}{2\sqrt{5}}$$

$$= \frac{5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{5\sqrt{5}}{2 \cdot 5}$$

$$= \frac{5\sqrt{5}}{10}$$

$$= \frac{\sqrt{5}}{2}$$

$$18.3 \quad \frac{(2+\sqrt{3})(2-\sqrt{3})}{\sqrt{12}}$$

$$= \frac{4-3}{\sqrt{4 \cdot 3}}$$

$$= \frac{1}{2\sqrt{3}}$$

$$= \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2 \cdot 3} = \frac{\sqrt{3}}{6}$$

$$18.2 \quad \frac{\sqrt{5}}{2+\sqrt{5}}$$

$$= \frac{\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{\sqrt{5}(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$= \frac{2\sqrt{5}-5}{4-5}$$

$$= \frac{2\sqrt{5}-5}{-1}$$

$$= -2\sqrt{5} + 5$$

$$18.4 \quad \frac{3}{9-\sqrt{18}}$$

$$= \frac{3}{9-\sqrt{9 \cdot 2}}$$

$$= \frac{3}{9-3\sqrt{2}}$$

$$= \frac{3}{3(3-\sqrt{2})}$$

$$= \frac{1}{(3-\sqrt{2})} \times \frac{(3+\sqrt{2})}{(3+\sqrt{2})}$$

$$= \frac{3+\sqrt{2}}{9-2}$$

$$= \frac{3+\sqrt{2}}{7}$$

$$\begin{aligned}
18.5 \quad & \frac{8}{\sqrt[3]{2}} \\
&= \frac{8}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \\
&= \frac{8(\sqrt[3]{2})^2}{(\sqrt[3]{2})^3} \\
&= \frac{8\sqrt[3]{2^2}}{2} \\
&= 4\sqrt[3]{4}
\end{aligned}$$

$$\begin{aligned}
18.7 \quad & \frac{2}{(2-\sqrt{3})^2} \\
&= \frac{2}{4-4\sqrt{3}+3} \\
&= \frac{2}{7-4\sqrt{3}} \\
&= \frac{2}{(7-4\sqrt{3})} \times \frac{(7+4\sqrt{3})}{(7+4\sqrt{3})}
\end{aligned}$$

$$\begin{aligned}
20. \quad LK &= \frac{\sqrt{x}}{x} + \frac{y}{\sqrt{x}} & RK &= \frac{\sqrt{x}(y+1)}{x} \\
LK &= \frac{\sqrt{x}}{x} + \frac{y}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} \\
&= \frac{\sqrt{x}}{x} + \frac{y \cdot \sqrt{x}}{x} \\
&= \frac{\sqrt{x} + y \cdot \sqrt{x}}{x} \\
&= \frac{\sqrt{x}(1+y)}{x} \\
&= RK
\end{aligned}$$

$$\begin{aligned}
18.6 \quad & \frac{1-\sqrt{8}}{(5+\sqrt{2})(2-\sqrt{2})} \\
&= \frac{1-2\sqrt{2}}{10-5\sqrt{2}+2\sqrt{2}-2} \\
&= \frac{1-2\sqrt{2}}{8-3\sqrt{2}} \times \frac{(8+3\sqrt{2})}{(8+3\sqrt{2})} \\
&= \frac{(1-2\sqrt{2})(8+3\sqrt{2})}{(8-3\sqrt{2})(8+3\sqrt{2})} \\
&= \frac{8+3\sqrt{2}-16\sqrt{2}-6.2}{64-9.2} \\
&= \frac{-4-13\sqrt{2}}{46}
\end{aligned}$$

$$\begin{aligned}
19. \quad & \frac{\sqrt[3]{16}-\sqrt[3]{54}}{\sqrt[3]{2}} \\
&= \frac{\sqrt[3]{8 \cdot 2} - \sqrt[3]{27 \cdot 2}}{\sqrt[3]{2}} \\
&= \frac{2\sqrt[3]{2} - 3\sqrt[3]{2}}{\sqrt[3]{2}} \\
&= \frac{-\sqrt[3]{2}}{\sqrt[3]{2}} \\
&= -1 \\
&= \frac{2(7+4\sqrt{3})}{(7-4\sqrt{3})(7+4\sqrt{3})} \\
&= \frac{14+8\sqrt{3}}{49-16.3} \\
&= \frac{14+8\sqrt{3}}{1} \\
&= 14 + 8\sqrt{3}
\end{aligned}$$

Hoofstuk 1

Eksponentes en Wortel

Antwoorde 2: Rasionale eksponente

$$1. 4^{\frac{1}{2}} = \left(2^{\frac{2}{1}}\right)^{\frac{1}{2}} = 2$$

$$2. \left(\frac{1}{16}\right)^{\frac{1}{4}} = \left(\frac{1}{2^4}\right)^{\frac{1}{4}} = (2^{-4})^{\frac{1}{4}} = 2^{-1} = \frac{1}{2}$$

$$\begin{aligned} 3. \left(\frac{81}{x^{-4}}\right)^{\frac{3}{4}} &= \left(\frac{3^4}{x^{-4}}\right)^{\frac{3}{4}} \\ &= \frac{\left(3^{\frac{4}{1}}\right)^{\frac{3}{4}}}{\left(x^{\frac{-4}{1}}\right)^{\frac{3}{4}}} \\ &= \frac{3^3}{x^{-3}} \\ &= 27x^3 \end{aligned}$$

$$\begin{aligned} 4. (16y^{16})^{\frac{1}{4}} &= (2^4y^{16})^{\frac{1}{4}} \\ &= \left(2^{\frac{4}{1}}\right)^{\frac{1}{4}} \cdot \left(y^{\frac{16}{1}}\right)^{\frac{1}{4}} \\ &= 2^1 \cdot y^4 \\ &= 2y^4 \end{aligned}$$

$$\begin{aligned} 5. 4x^{\frac{1}{2}} \div (4x)^{-\frac{1}{2}} &= \frac{4x^{\frac{1}{2}}}{(4x)^{-\frac{1}{2}}} \\ &= \frac{2^2 \cdot x^{\frac{1}{2}}}{(2^2 \cdot x)^{-\frac{1}{2}}} \\ &= \frac{2^2 \cdot x^{\frac{1}{2}}}{2^{-1} \cdot x^{-\frac{1}{2}}} \\ &= 2^2 \cdot x^{\frac{1}{2}} \cdot 2^1 \cdot x^{\frac{1}{2}} \\ &= 2^3 \cdot x^{\frac{2}{2}} \\ &= 8x \end{aligned}$$

$$\begin{aligned} 6. (0,008)^{-\frac{2}{3}} &= \left(\frac{8}{1000}\right)^{-\frac{2}{3}} \\ &= \left(\frac{1}{125}\right)^{-\frac{2}{3}} \\ &= \left(\frac{1}{5^3}\right)^{-\frac{2}{3}} \\ &= \left(5^{\frac{-3}{1}}\right)^{-\frac{2}{3}} \\ &= 5^2 \\ &= 25 \end{aligned}$$

$$\begin{aligned} 7. 32^{\frac{3}{5}} \cdot \left(\frac{3}{2}\right)^2 \div \sqrt{\frac{81}{16}} &= \left(2^{\frac{5}{1}}\right)^{\frac{3}{5}} \cdot \frac{3^2}{2^2} \div \frac{9}{4} \\ &= \frac{2^3}{1} \cdot \frac{3^2}{2^2} \times \frac{2^2}{3^2} \\ &= 8 \end{aligned}$$

$$\begin{aligned} 8. 2(9+16)^{\frac{1}{2}} - 5 \cdot \sqrt{\frac{1}{25}} &= 2(25)^{\frac{1}{2}} - \frac{5}{1} \cdot \frac{1}{5} \\ &= 2\left(5^{\frac{2}{1}}\right)^{\frac{1}{2}} - \frac{5}{1} \cdot \frac{1}{5} \\ &= 2 \cdot 5 - 1 \\ &= 9 \end{aligned}$$

$$\begin{aligned}
9. & \left[\left(a^{-\frac{2}{3}} \cdot b^{\frac{4}{3}} \right)^{\frac{1}{2}} \cdot \left(a^{\frac{1}{2}} \cdot b^{-\frac{1}{4}} \right)^{-\frac{4}{3}} \right]^{-1} \\
&= \left[a^{-\frac{1}{3}} \cdot b^{\frac{2}{3}} \cdot a^{-\frac{2}{3}} \cdot b^{\frac{1}{3}} \right]^{-1} \\
&= \left[a^{-\frac{1}{3}-\frac{2}{3}} \cdot b^{\frac{2}{3}+\frac{1}{3}} \right]^{-1} \\
&= \left[a^{-\frac{3}{3}} \cdot b^{\frac{3}{3}} \right]^{-1} \\
&= [a^{-1} \cdot b^1]^{-1} \\
&= a \cdot b^{-1} \\
&= \frac{a}{b}
\end{aligned}$$

$$\begin{aligned}
11. & 4^{\frac{3}{2}} + 8^{\frac{1}{3}} - (0,5)^{-2} \\
&= \left(2^{\frac{2}{1}} \right)^{\frac{3}{2}} + \left(2^{\frac{3}{1}} \right)^{\frac{1}{3}} - \left(\frac{1}{2} \right)^{-2} \\
&= 2^3 + 2^1 - (2^{-1})^{-2} \\
&= 8 + 2 - 2^2 \\
&= 8 + 2 - 4 \\
&= 6
\end{aligned}$$

$$\begin{aligned}
13. & \left(\frac{\sqrt{a}}{\sqrt{a^{-3}}} \right)^{-\frac{1}{2}} \\
&= \left(\frac{a^{\frac{1}{2}}}{a^{-\frac{3}{2}}} \right)^{-\frac{1}{2}} \\
&= \left(a^{\frac{1}{2}} \cdot a^{\frac{3}{2}} \right)^{-\frac{1}{2}} \\
&= \left(a^{\frac{4}{2}} \right)^{-\frac{1}{2}} \\
&= a^{-1} \\
&= \frac{1}{a}
\end{aligned}$$

$$\begin{aligned}
16. & \left(x^{\frac{1}{6}} + 1 \right) \left(x^{\frac{1}{6}} - 1 \right) \\
&= x^{\frac{1}{6}} \left(x^{\frac{1}{6}} \right) - 1 \quad \text{EBBA se kortpad} \\
&= x^{\frac{2}{6}} - 1 \\
&= x^{\frac{1}{3}} - 1
\end{aligned}$$

$$\begin{aligned}
10. & \sqrt{a^{\frac{1}{2}}} \cdot (bc)^{-\frac{1}{2}} \cdot \sqrt{c^3} \cdot b^{\frac{1}{2}} \cdot \left(a^{\frac{1}{4}} \right)^3 \\
&= a^{\left(\frac{1}{2}\right) \div 2} \cdot b^{-\frac{1}{2}} \cdot c^{-\frac{1}{2}} \cdot c^{\frac{3}{2}} \cdot b^{\frac{1}{2}} \cdot \left(a^{\frac{1}{4}} \right)^{\frac{3}{1}} \\
&= a^{\left(\frac{1}{2}\right) \times \frac{1}{2}} \cdot b^{-\frac{1}{2}} \cdot c^{-\frac{1}{2}} \cdot c^{\frac{3}{2}} \cdot b^{\frac{1}{2}} \cdot a^{\frac{3}{4}} \\
&= a^{\frac{1}{4}} \cdot b^{-\frac{1}{2}} \cdot c^{-\frac{1}{2}} \cdot c^{\frac{3}{2}} \cdot b^{\frac{1}{2}} \cdot a^{\frac{3}{4}} \\
&= a^{\frac{1}{4}+\frac{3}{4}} \cdot b^{-\frac{1}{2}+\frac{1}{2}} \cdot c^{-\frac{1}{2}+\frac{3}{2}} \\
&= a^1 \cdot b^0 \cdot c^1 \\
&= a^1 \cdot 1 \cdot c^1 \\
&= ac
\end{aligned}$$

$$\begin{aligned}
12. & \frac{x^{8n-2} \cdot \left(xy^{\frac{1}{3}} \right)^{-6n} \cdot x^2}{\left(\frac{1}{xy} \right)^{3n} \cdot (y^{-1})^n} \\
&= \frac{x^{8n-2} \cdot x^{-6n} \left(y^{\frac{1}{3}} \right)^{\frac{-6n}{1}} \cdot x^2}{(x^{-1} \cdot y^{-1})^{3n} \cdot (y^{-1})^n} \\
&= \frac{x^{8n-2} \cdot x^{-6n} \cdot y^{-2n} \cdot x^2}{x^{-3n} \cdot y^{-3n} \cdot y^{-n}} \\
&= x^{8n-2} \cdot x^{-6n} \cdot y^{-2n} \cdot x^2 \cdot x^{3n} \cdot y^{3n} \cdot y^n \\
&= x^{8n-2-6n+2+3n} y^{-2n+3n+n} \\
&= x^{5n} y^{2n}
\end{aligned}$$

$$\begin{aligned}
14. & \left(x^{\frac{1}{3}} - y^{\frac{2}{3}} \right)^2 \quad (2 \text{ terme})^2 \text{ gee } 3 \text{ terme} \\
&= \left(x^{\frac{1}{3}} \right) \left(x^{\frac{1}{3}} \right) - 2 \left(x^{\frac{1}{3}} \right) \left(y^{\frac{2}{3}} \right) + \left(y^{\frac{2}{3}} \right) \left(y^{\frac{2}{3}} \right) \\
&= x^{\frac{1}{3}+\frac{1}{3}} - 2x^{\frac{1}{3}} y^{\frac{2}{3}} + y^{\frac{2}{3}+\frac{2}{3}} \\
&= x^{\frac{2}{3}} - 2x^{\frac{1}{3}} y^{\frac{2}{3}} + y^{\frac{4}{3}}
\end{aligned}$$

$$\begin{aligned}
15. & x^{\frac{1}{2}} \left(x^{\frac{1}{2}} + x - 1 \right) \\
&= x^{\frac{1}{2}} \left(x^{\frac{1}{2}} \right) + x^{\frac{1}{2}} (x^1) - x^{\frac{1}{2}} (1) \\
&= x^{\frac{1}{2}+\frac{1}{2}} + x^{\frac{1}{2}+1} - x^{\frac{1}{2}} \\
&= x^1 + x^{\frac{3}{2}} - x^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
17. \quad & (a^1 + 2a^{-1})(a^2 - 2 + 4a^{-2}) \\
&= a(a^2) + a(-2) + a(4a^{-2}) + 2a^{-1}(a^2) + 2a^{-1}(-2) + 2a^{-1}(4a^{-2}) \\
&= a^3 - 2a + 4a^{-1} + 2a - 4a^{-1} + 8a^{-3} \\
&= a^3 + 8a^{-3} \\
&= a^3 + \frac{8}{a^3}
\end{aligned}$$

$$18. \quad x^{\frac{3}{2}}(x^{-\frac{1}{2}} - x^{\frac{1}{2}})$$

$$\begin{aligned}
&= x^{\frac{3}{2}}(x^{-\frac{1}{2}}) - x^{\frac{3}{2}}(x^{\frac{1}{2}}) \\
&= x^{\frac{3}{2}-\frac{1}{2}} - x^{\frac{3}{2}+\frac{1}{2}} \\
&= x^{\frac{2}{2}} - x^{\frac{4}{2}} \\
&= x - x^2
\end{aligned}$$

$$19. \quad (2x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2$$

$$\begin{aligned}
&= (2x^{\frac{1}{2}})(2x^{\frac{1}{2}}) + 2(2x^{\frac{1}{2}})(x^{-\frac{1}{2}}) + (x^{-\frac{1}{2}})(x^{-\frac{1}{2}}) \\
&= 4x^{\frac{1}{2}+\frac{1}{2}} + 4x^{\frac{1}{2}-\frac{1}{2}} + x^{-\frac{1}{2}-\frac{1}{2}} \\
&= 4x^1 + 4x^0 + x^{-1} \\
&= 4x + 4 + \frac{1}{x} \\
&= 4x + 4 + \frac{1}{x}
\end{aligned}$$

$$20. \quad (x^{\frac{1}{2}} - y^{\frac{1}{2}})^2 + 2x^{\frac{1}{2}}y^{\frac{1}{2}}$$

$$\begin{aligned}
&= (x^{\frac{1}{2}})(x^{\frac{1}{2}}) - 2 \cdot x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} + (y^{\frac{1}{2}})(y^{\frac{1}{2}}) + 2x^{\frac{1}{2}}y^{\frac{1}{2}} \\
&= x^{\frac{1}{2}+\frac{1}{2}} + y^{\frac{1}{2}+\frac{1}{2}} \\
&= x^1 + y^1 \\
&= x + y
\end{aligned}$$

Hoofstuk 1

Eksponentes en Wortel

Antwoorde 3: Uitdrukings

1. $\frac{49 \cdot 2^{\frac{1}{2}x+2} \cdot 3^x}{21^{x+3}}$ NB Slegs faktore - alle faktore na teller
- $= \frac{(7^2)^{\frac{1}{2}x+2} \cdot 3^x}{(3 \cdot 7)^{x+3}}$ maak basisse priem
- $= \frac{7^{x+4} \cdot 3^x}{3^{x+3} \cdot 7^{x+3}}$ verwyder hakies - vermenigvuldig eksponente uit
- $= 7^{x+4} \cdot 3^x \cdot 3^{-x-3} \cdot 7^{-x-3}$
- $= 7^{x+4-x-3} \cdot 3^{x-x-3}$
- $= 7^1 \cdot 3^{-3}$ maak negatiewe eksponente possitief
- $= \frac{7}{3^3}$
- $= \frac{7}{27}$
2. $\frac{4^{r+1} 36^{-r-1}}{45^{-r+1} 5^{r+1} 81^{-1}}$ NB Slegs faktore - alle faktore na teller
- $= \frac{(2^2)^{r+1} (2^2 \cdot 3^2)^{-r-1}}{(5 \cdot 3^2)^{-r+1} 5^{r+1} (3^4)^{-1}}$ $36 = 4 \times 9 = 2^2 \cdot 3^2$; $45 = 5 \times 9 = 5 \cdot 3^2$
- $= \frac{2^{2r+2} \cdot 2^{-2r-2} \cdot 3^{-2r-2}}{5^{-r+1} \cdot 3^{-2r+2} \cdot 5^{r+1} \cdot 3^{-4}}$ $(2^2 \cdot 3^2)^{-r-1} = (2^2)^{-r-1} (3^2)^{-r-1}$
- $= 2^{2r+2} \cdot 2^{-2r-2} \cdot 3^{-2r-2} \cdot 5^{r-1} \cdot 3^{2r-2} \cdot 5^{-r-1} \cdot 3^4$
- $= 2^0 \cdot 3^0 \cdot 5^{-2}$
- $= 1 \cdot 1 \cdot 5^{-2}$
- $= \frac{1}{5^2}$
- $= \frac{1}{25}$
3. $\frac{2^{-6y} \cdot 25^{-3}}{125^{y-2} \cdot 20^{-3y} \cdot 8}$ NB Slegs faktore - alle faktore na teller
- $= \frac{2^{-6y} \cdot (5^2)^{-3}}{(5^3)^{y-2} \cdot (2^2 \cdot 5)^{-3y} \cdot (2^3)}$ $20 = 4 \times 5 = 2^2 \cdot 5$
- $= \frac{2^{-6y} \cdot 5^{-6}}{5^{3y-6} \cdot 2^{-6y} \cdot 5^{-3y} \cdot 2^3}$
- $= 2^{-6y} \cdot 5^{-6} \cdot 5^{-3y+6} \cdot 2^{6y} \cdot 5^{3y} \cdot 2^{-3}$
- $= 2^{-3} \cdot 5^0$
- $= \frac{1}{2^3} \cdot 1$
- $= \frac{1}{8}$

4. $\sqrt[4]{\frac{3^a \cdot 9^{a+1}}{27^{a+2}}}$ NB Slegs faktore - alle faktore na teller

$= \sqrt[4]{\frac{3^a \cdot (3^2)^{a+1}}{(3^3)^{a+2}}}$ basisse priem

$= \sqrt[4]{\frac{3^a \cdot 3^{2a+2}}{3^{3a+6}}}$ verwyder hakies

$= \sqrt[4]{3^a \cdot 3^{2a+2} \cdot 3^{-3a-6}}$

$= \sqrt[4]{3^{a+2a+2-3a-6}}$

$= \sqrt[4]{3^{-4}}$ werk wortel uit na magsvorm

$= 3^{-\frac{4}{4}}$

$= 3^{-1}$

$= \frac{1}{3}$

5. $\frac{(a+x)^{\frac{1}{2}} \cdot \sqrt{a^2-x^2}}{(a-x)^{-\frac{1}{2}}}$ NB Slegs faktore - alle faktore na teller

$= \frac{(a+x)^{\frac{1}{2}} \cdot \sqrt{(a-x)(a+x)}}{(a-x)^{-\frac{1}{2}}}$ faktoriseer

$= \frac{(a+x)^{\frac{1}{2}} \cdot \sqrt{(a-x)} \cdot \sqrt{(a+x)}}{(a-x)^{-\frac{1}{2}}}$ vat elke faktor se wortel apart

$= (a+x)^{\frac{1}{2}} \cdot (a-x)^{\frac{1}{2}} \cdot (a+x)^{\frac{1}{2}} \cdot (a-x)^{\frac{1}{2}}$ skryf wortels oor in magsvorm

$= (a+x)^{\frac{1}{2}+\frac{1}{2}} \cdot (a-x)^{\frac{1}{2}+\frac{1}{2}}$ vermenigvuldig magte wat dieselfde basisse het

$= (a+x)^1 \cdot (a-x)^1$

$= a^2 - x^2$

6. $\frac{3 \cdot 9^{x+1} + 3^{2x+4}}{3^x \cdot 3^{x-1} - 9^x}$ NB Ook terme - Faktoriseer

$= \frac{3 \cdot (3^2)^{x+1} + 3^{2x+4}}{3^x \cdot 3^{x-1} - (3^2)^x}$ Basisse priem

$= \frac{3 \cdot 3^{2x+2} + 3^{2x+4}}{3^x \cdot 3^{x-1} - 3^{2x}}$ Verwyder hakies

$= \frac{3 \cdot 3^{2x} \cdot 3^2 + 3^{2x} \cdot 3^4}{3^x \cdot 3^x \cdot 3^{-1} - 3^{2x}}$ Gee elke eksp sy eie basis

$= \frac{3^{2x} \cdot 3^3 + 3^{2x} \cdot 3^4}{3^{2x} \cdot 3^{-1} - 3^{2x}}$

$= \frac{3^{2x} \cdot (3^3 + 3^4)}{3^{2x} \cdot (3^{-1} - 1)}$ Haal 3^{2x} as gem faktor uit en kanselleer

$= \frac{27+81}{\frac{1}{3}-1}$

$= \frac{108}{-\frac{2}{3}}$

$= 108 \times \left(\frac{-3}{2}\right)$

$= -162$

$$\begin{aligned}
7. \quad & \frac{(3^n)^2 + 3^{2n-1}}{9^n} \\
&= \frac{3^{2n} + 3^{2n-1}}{(3^2)^n} \\
&= \frac{3^{2n} + 3^{2n} \cdot 3^{-1}}{3^{2n}} \\
&= \frac{3^{2n} \cdot (1 + 3^{-1})}{3^{2n}} \\
&= 1 + \frac{1}{3} \\
&= \frac{4}{3}
\end{aligned}$$

NB Ook terme - Faktoriseer

Gee elke eksp sy eie basis

Haal 3^{2n} as gem faktor uit en kanselleer

$$\begin{aligned}
9. \quad & \frac{2^{x+1} + 2 \cdot 3^{y-1}}{2^{2+x} + 4 \cdot 3^{y-1}} \\
&= \frac{2^x \cdot 2^1 + 2 \cdot 3^y \cdot 3^{-1}}{2^2 \cdot 2^x + 2^2 \cdot 3^y \cdot 3^{-1}} \\
&= \frac{2^1 \cdot (2^x + 3^y \cdot 3^{-1})}{2^2 \cdot (2^x + 3^y \cdot 3^{-1})} \\
&= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
8. \quad & \left(\frac{16^{x+1} - 2 \cdot 4^{2x+1}}{9 \cdot 4^x - 2^{2x}} \right)^{\frac{1}{x}} \\
&= \left[\frac{(2^4)^{x+1} - 2 \cdot (2^2)^{2x+1}}{9 \cdot (2^2)^x - 2^{2x}} \right]^{\frac{1}{x}} \\
&= \left[\frac{2^{4x+4} - 2 \cdot 2^{4x+2}}{9 \cdot 2^{2x} - 2^{2x}} \right]^{\frac{1}{x}} \\
&= \left[\frac{2^{4x} \cdot 2^4 - 2 \cdot 2^{4x} \cdot 2^2}{9 \cdot 2^{2x} - 2^{2x}} \right]^{\frac{1}{x}} \\
&= \left[\frac{2^{4x} (2^4 - 2 \cdot 2^2)}{2^{2x} (9 - 1)} \right]^{\frac{1}{x}} \\
&= \left[\frac{2^{4x} (8)}{2^{2x} (8)} \right]^{\frac{1}{x}} \\
&= \left[\frac{2^{4x}}{2^{2x}} \right]^{\frac{1}{x}} \\
&= [2^{4x} \cdot 2^{-2x}]^{\frac{1}{x}} \\
&= \left[2^{\frac{2x}{1}} \right]^{\frac{1}{x}} \\
&= 2^2 \\
&= 4
\end{aligned}$$

$$\begin{aligned}
10. \quad & \frac{\sqrt{x} - \sqrt{y}}{x - y} \\
&= \frac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{x^1 - y^1} \\
&= \frac{\left(x^{\frac{1}{2}} - y^{\frac{1}{2}} \right)}{\left(x^{\frac{1}{2}} - y^{\frac{1}{2}} \right) \left(x^{\frac{1}{2}} + y^{\frac{1}{2}} \right)} \\
&= \frac{1}{\left(x^{\frac{1}{2}} + y^{\frac{1}{2}} \right)}
\end{aligned}$$

OF

$$\begin{aligned}
& \frac{(\sqrt{x} - \sqrt{y})}{(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})} \\
&= \frac{1}{(\sqrt{x} + \sqrt{y})}
\end{aligned}$$

$$\begin{aligned}
12. \quad & \frac{x^{-2}-y^{-2}}{x^{-2} \cdot y \cdot y^{-2} \cdot x} \\
&= \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\left(\frac{1}{x^2}\right) \frac{y}{1} - \left(\frac{1}{y^2}\right) \frac{x}{1}} \\
&= \frac{\frac{(y^2-x^2)}{x^2 y^2}}{\frac{y}{x^2} - \frac{x}{y^2}} \\
&= \frac{\frac{(y^2-x^2)}{x^2 y^2}}{\frac{(y^3-x^3)}{x^2 y^2}} \\
&= \frac{(y-x)(y+x)}{x^2 y^2} \times \frac{x^2 y^2}{(y-x)(y^2+xy+x^2)} \\
&= \frac{y+x}{y^2+xy+x^2}
\end{aligned}$$

$$\begin{aligned}
13. \quad & \frac{2^{2x}-1}{2^x-1} \\
&= \frac{(2^x-1)(2^x+1)}{(2^x-1)} \\
&= (2^x + 1)
\end{aligned}$$

$$\begin{aligned}
15. \quad & \frac{2^{2x}-2^x}{2^x-1} \\
&= \frac{2^x \cdot 2^x - 2^x}{2^x-1} \\
&= \frac{2^x(2^x-1)}{(2^x-1)} \\
&= 2^x
\end{aligned}$$

$$\begin{aligned}
11. \quad & \frac{x^{\frac{3}{2}} - x^{-\frac{1}{2}}}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}} \\
&= \frac{\frac{x^{\frac{3}{2}}}{1} - \frac{1}{x^{\frac{1}{2}}}}{\frac{x^{\frac{1}{2}}}{1} + \frac{1}{x^{\frac{1}{2}}}} \\
&= \frac{\frac{x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} - 1}{x^{\frac{1}{2}}}}{\frac{x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} + 1}{x^{\frac{1}{2}}}} \\
&= \frac{\frac{x^{\frac{3}{2} + \frac{1}{2}} - 1}{x^{\frac{1}{2}}}}{\frac{x^{\frac{1}{2} + \frac{1}{2}} + 1}{x^{\frac{1}{2}}}} \\
&= \frac{\frac{x^2 - 1}{x^{\frac{1}{2}}}}{\frac{x^1 + 1}{x^{\frac{1}{2}}}} \\
&= \frac{x^2 - 1}{x^{\frac{1}{2}}} \times \frac{x^{\frac{1}{2}}}{x+1} \\
&= \frac{(x-1)(x+1)}{x^{\frac{1}{2}}} \times \frac{x^{\frac{1}{2}}}{(x+1)} \\
&= x - 1
\end{aligned}$$

$$\begin{aligned}
14. \quad & \frac{16^x - 9}{4^x + 3} \\
&= \frac{4^{2x} - 9}{4^x + 3} \\
&= \frac{(4^x - 3)(4^x + 3)}{(4^x + 3)} \\
&= (4^x - 3)
\end{aligned}$$

$$\begin{aligned}
16. \quad & \frac{2^{2t} + 2^{t+1} - 15}{2^t + 5} \\
&= \frac{2^{2t} + 2^t \cdot 2^1 - 15}{(2^t + 5)} \\
&= \frac{1 \cdot 2^{2t} + 2 \cdot 2^t - 15}{(2^t + 5)} \\
&= \frac{(2^t - 3)(2^t + 5)}{(2^t + 5)} \\
&= 2^t - 3
\end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{3^{2a+2} \cdot 3^{a+1} - 7}{3^a - 1} \\
 &= \frac{3^{2a+2} \cdot 3^a \cdot 3^1 - 7}{3^a - 1} \\
 &= \frac{3^{2a+6} \cdot 3^a - 7}{(3^a - 1)} \\
 &= \frac{(3^a + 7)(3^a - 1)}{(3^a - 1)}
 \end{aligned}$$

$$= 3^a + 7$$

$$\begin{aligned}
 19. \quad & \frac{p^{-4} \sqrt{p} + 3}{p^{\frac{1}{2}} - 3} \\
 &= \frac{p^{-4} p^{\frac{1}{2}} + 3}{p^{\frac{1}{2}} - 3} \\
 &= \frac{\left(p^{\frac{1}{2}} - 1\right) \left(p^{\frac{1}{2}} - 3\right)}{\left(p^{\frac{1}{2}} - 3\right)}
 \end{aligned}$$

$$= p^{\frac{1}{2}} - 1$$

$$\begin{aligned}
 18. \quad & \frac{x - 2x^{\frac{1}{2}} + 1}{x^{\frac{1}{2}} - 1} \\
 &= \frac{\left(x^{\frac{1}{2}} - 1\right) \left(x^{\frac{1}{2}} - 1\right)}{\left(x^{\frac{1}{2}} - 1\right)}
 \end{aligned}$$

$$= x^{\frac{1}{2}} - 1$$

$$20. \quad \frac{a^{\frac{3}{2}} + 5a^{\frac{1}{2}} + 6a^{-\frac{1}{2}}}{a^2 - 4}$$

$$= \frac{a^{\frac{3}{2}} + 5a^{\frac{1}{2}} + \frac{6}{a^{\frac{1}{2}}}}{a^2 - 4}$$

$$= \frac{a^{\frac{3}{2}} a^{\frac{1}{2}} + 5a^{\frac{1}{2}} a^{\frac{1}{2}} + \frac{6}{a^{\frac{1}{2}}}}{a^2 - 4}$$

$$= \frac{\frac{4}{a^{\frac{1}{2}}} + 5a^{\frac{2}{2}} + 6}{\frac{1}{a^{\frac{1}{2}}}}$$

$$= \frac{a^2 + 5a + 6}{a^{\frac{1}{2}}} \times \frac{1}{a^2 - 4}$$

$$= \frac{(a+3)(a+2)}{a^{\frac{1}{2}}} \times \frac{1}{(a-2)(a+2)}$$

$$= \frac{(a+3)}{a^{\frac{1}{2}} \cdot (a-2)}$$

Hoofstuk 1

Eksponentes en Wortel

Antwoorde 4: Vergelykings

1. $2^x - 32 = 0$

$$2^x = 32$$

$$2^x = 2^5$$

$$x = 5$$

2. $\left(\frac{1}{2}\right)^x = 4$

$$(2^{-1})^x = 2^2$$

$$2^{-x} = 2^2$$

$$-x = 2$$

$$x = -2$$

3. $3 \cdot 2^x = 24$

$$2^x = 8 \quad \div 3$$

$$2^x = 2^3$$

$$x = 3$$

4. $2^x = 64^{x+1}$

$$2^x = (2^6)^{x+1}$$

$$2^x = 2^{6x+6}$$

$$x = 6x + 6$$

$$-6 = 5x$$

$$-\frac{6}{5} = x$$

5. $2^x = 8^y \dots (1)$

en

$3^{x-1} = 9^y \dots (2)$

$$2^x = (2^3)^y$$

$$3^{x-1} = (3^2)^y$$

$$2^x = 2^{3y}$$

$$3^{x-1} = 3^{2y}$$

$$x = 3y \dots (1)$$

$$x - 1 = 2y \dots (2)$$

Stel x in $x - 1 = 2y \dots (2)$

$$3y - 1 = 2y$$

$$y = 1$$

Stel y in $x = 3y \dots (1)$

$$x = 3(1)$$

$$x = 3$$

6. $4 \cdot 2^y = \frac{2^{x^2}}{2^1} \dots (1)$

en

$3^y = \frac{1}{9^x} \dots (2)$

$$(2^2) \cdot 2^y = 2^{x^2} \cdot 2^{-1}$$

$$3^y = 9^{-x}$$

$$2^{2+y} = 2^{x^2-1}$$

$$3^y = (3^2)^{-x}$$

$$2 + y = x^2 - 1$$

$$3^y = 3^{-2x}$$

$$3 + y = x^2 \dots (1)$$

$$y = -2x \dots (2)$$

Stel y in $3 + y = x^2 \dots (1)$

$$3 + (-2x) = x^2$$

Stel x terug in $y = -2x \dots (2)$

$$0 = x^2 + 2x - 3$$

$$y = -2(-3) \quad \text{of} \quad y = -2(1)$$

$$0 = (x+3)(x-1)$$

$$y = 6 \quad \quad \quad y = -2$$

$$x = -3 \quad \text{of} \quad x = 1$$

$$8. \quad 3^{x+2} - 3^{x-1} - 78 = 0$$

$$3^{x+2} - 3^{x-1} = 78$$

$$3^x \cdot 3^2 - 3^x \cdot 3^{-1} = 78$$

$$3^x(3^2 - 3^{-1}) = 78$$

$$3^x\left(9 - \frac{1}{3}\right) = 78$$

$$3^x\left(\frac{26}{3}\right) = 78$$

$$\frac{3^x\left(\frac{26}{3}\right)}{\left(\frac{26}{3}\right)} = \frac{78}{\left(\frac{26}{3}\right)}$$

$$3^x = \frac{78}{1} \times \frac{3}{26} = 9$$

$$3^x = 3^2$$

$$x = 2$$

$$9. \quad 2^{x+2} - 2^{x-1} = 28$$

$$2^x \cdot 2^2 - 2^x \cdot 2^{-1} = 28$$

$$2^x\left(4 - \frac{1}{2}\right) = 28$$

$$2^x\left(\frac{7}{2}\right) = 28$$

$$\frac{2^x\left(\frac{7}{2}\right)}{\left(\frac{7}{2}\right)} = \frac{28}{\left(\frac{7}{2}\right)}$$

$$2^x = \frac{28}{1} \times \frac{2}{7} = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$11. \quad 3^{2x+2} - 10 \cdot 3^x + 1 = 0$$

$$3^{2x} \cdot 3^2 - 10 \cdot 3^x + 1 = 0$$

$$9 \cdot 3^{2x} - 10 \cdot 3^x + 1 = 0$$

$$(9 \cdot 3^x - 1)(1 \cdot 3^x - 1) = 0$$

$$9 \cdot 3^x = 1 \quad \text{of} \quad 3^x = 1$$

$$3^x = \frac{1}{9} \quad 3^x = 3^0$$

$$3^x = 3^{-2} \quad x = 0$$

$$x = -2$$

$$7. \quad 5^{x+1} = \frac{7}{25} - 2 \cdot 5^x$$

$$5^{x+1} + 2 \cdot 5^x = \frac{7}{25}$$

$$5^x \cdot 5^1 + 2 \cdot 5^x = \frac{7}{25}$$

$$5^x(5^1 + 2) = \frac{7}{25}$$

$$5^x(7) = \frac{7}{25}$$

$$\frac{5^x(7)}{7} = \frac{7}{25} \div 7$$

$$5^x = \frac{7}{25} \times \frac{1}{7}$$

$$5^x = \frac{1}{25}$$

$$5^x = 5^{-2}$$

$$x = -2$$

$$10. \quad 2^{x+1} + 2^x - 2^{x-1} = 10$$

$$2^x \cdot 2^1 + 2^x - 2^x \cdot 2^{-1} = 10$$

$$2^x(2^1 + 1 - 2^{-1}) = 10$$

$$2^x\left(\frac{2}{1} + \frac{1}{1} - \frac{1}{2}\right) = 10$$

$$2^x\left(\frac{5}{2}\right) = 10$$

$$\frac{2^x\left(\frac{5}{2}\right)}{\frac{5}{2}} = \frac{10}{\frac{5}{2}}$$

$$2^x = \frac{10}{1} \times \frac{2}{5} = 4$$

$$2^x = 2^2$$

$$x = 2$$

$$12. \quad 9 + 8 \cdot 3^x = 3^{2x}$$

$$0 = 1 \cdot 3^{2x} - 8 \cdot 3^x - 9$$

$$0 = (1 \cdot 3^x + 1)(1 \cdot 3^x - 9)$$

$$3^x = -1 \quad \text{of} \quad 3^x = 9$$

$$\text{geen oplossing} \quad 3^x = 3^2$$

$$x = 2$$

$$13. \quad 3^{2x} = 3 - 26 \cdot 3^{x-1}$$

$$3^{2x} = 3 - 26 \cdot 3^x \cdot 3^{-1}$$

$$3^{2x} = 3 - \frac{26}{1} \cdot \frac{1}{3} \cdot 3^x$$

$$3^{2x} = 3 - \frac{26}{3} \cdot 3^x$$

$$3^{2x} \times 3 = 3 \times 3 - \frac{26}{3} \cdot 3^x \times \frac{3}{1}$$

$$3 \cdot 3^{2x} = 9 - 26 \cdot 3^x$$

$$3 \cdot 3^{2x} + 26 \cdot 3^x - 9 = 0$$

$$(3 \cdot 3^x - 1)(1 \cdot 3^x + 9) = 0$$

$$3 \cdot 3^x = 1 \quad \text{of} \quad 3^x = -9$$

$$3^x = \frac{1}{3} \quad \text{geen oplossing}$$

$$3^x = 3^{-1}$$

$$x = -1$$

OF k -methode

$$3k^2 + 26k - 9 = 0$$

$$(3k - 1)(k + 9) = 0$$

$$k = \frac{1}{3} \quad \text{of} \quad k = -9$$

$$3^x = \frac{1}{3} \quad 3^x = -9$$

$$3^x = \frac{1}{3} \quad \text{geen oplossing}$$

$$14. \quad 3^{2-x} + 8 = 3^x \quad \text{een eksponent is } +x \text{ en ander een } -x \text{ - nie gemeenskaplike faktor}$$

$$3^2 \cdot 3^{-x} - 3^x + 8 = 0 \quad \text{moet kwadratiese wees, alles links en regs } = 0$$

$$\frac{9}{3^x} - 3^x + 8 = 0 \quad \text{maak eksponent positief}$$

$$\frac{9}{3^x} \times \frac{3^x}{1} - 3^x \times 3^x + 8 \times 3^x = 0 \times 3^x \quad \text{verwyder breuke}$$

$$9 - 3^{2x} + 8 \cdot 3^x = 0$$

OF k -methode

$$0 = 1 \cdot 3^{2x} - 8 \cdot 3^x - 9$$

$$0 = k^2 - 8k - 9$$

$$0 = (3^x - 9)(3^x + 1)$$

$$(k - 9)(k + 1) = 0$$

$$3^x = 9 \quad \text{of} \quad 3^x = -1$$

$$k = 9 \quad \text{of} \quad k = -1$$

$$3^x = 3^2$$

$$3^x = 9 \quad \text{of} \quad 3^x = -1$$

$$x = 2$$

$$x = 2 \quad \text{geen oplossing}$$

$$15. \quad 2^x + 16 \cdot 2^{-x} - 10 = 0$$

$$2^x + \frac{16}{2^x} - 10 = 0 \quad \text{maak neg eksp pos}$$

$$2^x \times 2^x + \frac{16}{2^x} \times \frac{2^x}{1} - 10 \times 2^x = 0 \quad \text{verwyder breuke}$$

$$2^{2x} + 16 - 10 \cdot 2^x = 0 \quad \text{kwadratiese 3 term, standaardvorm}$$

$$2^{2x} - 10 \cdot 2^x + 16 = 0$$

OF k -methode

$$(2^x - 8)(2^x - 2) = 0$$

$$k^2 - 10k + 16 = 0$$

$$2^x = 8 \quad \text{of} \quad 2^x = 2$$

$$(k - 8)(k - 2) = 0$$

$$2^x = 2^3 \quad 2^x = 2^1$$

$$k = 8 \quad \text{of} \quad k = 2$$

$$x = 3 \quad x = 1$$

$$2^x = 8 \quad \text{of} \quad 2^x = 2$$

$$x = 3 \quad x = 1$$

$$16. \quad 3^{x+1} - 4\sqrt{3^x} - 15 = 0$$

$$3 \cdot 3^x - 4 \cdot 3^{\frac{x}{2}} - 15 = 0 \quad \text{skryf wortel oor in magsvorm en split eksponente op}$$

$$(3 \cdot 3^{\frac{x}{2}} + 5)(3^{\frac{x}{2}} - 3) = 0 \quad \text{kwadratiese 3 term - eerste mag se eksponent van } x \text{ is dubbel 2de een}$$

$$3 \cdot 3^{\frac{x}{2}} + 5 = 0 \quad \text{of} \quad 3^{\frac{x}{2}} - 3 = 0$$

$$3 \cdot 3^{\frac{x}{2}} = -5 \quad 3^{\frac{x}{2}} = 3^1$$

$$3^{\frac{x}{2}} = -\frac{5}{3} \quad \frac{x}{2} = 1$$

$$\text{geen oplossing} \quad x = 2$$

$$17. \quad 3b^{-\frac{1}{2}} = \sqrt{3}$$

$$\frac{3b^{-\frac{1}{2}}}{3} = \frac{\sqrt{3}}{3}$$

$$b^{-\frac{1}{2}} = \frac{3^{\frac{1}{2}}}{3^1}$$

$$\frac{1}{b^{\frac{1}{2}}} = \frac{3^{\frac{1}{2}}}{3}$$

$$3^{\frac{1}{2}} b^{\frac{1}{2}} = 3$$

$$b^{\frac{1}{2}} = \frac{3}{3^{\frac{1}{2}}}$$

$$b^{\frac{1}{2}} = 3^1 \cdot 3^{-\frac{1}{2}}$$

$$\sqrt{b} = 3^{\frac{1}{2}}$$

$$(\sqrt{b})^2 = (3^{\frac{1}{2}})^2$$

$$b = 3$$

$$19. \quad x^{-\frac{2}{3}} = 16$$

$$\frac{1}{x^{\frac{2}{3}}} = \frac{16}{1}$$

draai beide kante om

$$x^{\frac{2}{3}} = \frac{1}{16}$$

skryf terug na wortelvorm en maak getal priem

$$\sqrt[3]{x^2} = \frac{1}{2^4}$$

$$\left(\sqrt[3]{x^2}\right)^3 = \left(\frac{1}{2^4}\right)^3$$

werk om x alleen te kry

$$x^2 = \frac{1}{2^{12}}$$

$$\sqrt{x^2} = \pm \sqrt{\frac{1}{2^{12}}}$$

onthou \pm as jy 'n ewemags wortel trek aan beide kante

$$x = \pm \frac{1}{2^6}$$

$$x = \pm \frac{1}{64}$$

$$18. \quad p^{-\frac{1}{3}} = \frac{1}{3}$$

$$\frac{1}{p^{\frac{1}{3}}} = \frac{1}{3}$$

draai beide kante om

$$p^{\frac{1}{3}} = 3$$

$$\sqrt[3]{p} = 3$$

$$\left(\sqrt[3]{p}\right)^3 = 3^3$$

$$p = 27$$

$$20. \quad x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + 2 = 0$$

$$(x^{\frac{1}{4}} - 2)(x^{\frac{1}{4}} - 1) = 0$$

$$x^{\frac{1}{4}} - 2 = 0 \quad \text{of} \quad x^{\frac{1}{4}} - 1 = 0$$

$$\sqrt[4]{x} = 2 \quad \sqrt[4]{x} = 1$$

$$(\sqrt[4]{x})^4 = 2^4 \quad (\sqrt[4]{x})^4 = 1^4$$

$$x = 16 \quad x = 1$$

$$22. \quad 4x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 6 = 0$$

$$(4 \cdot x^{\frac{1}{3}} - 3)(1 \cdot x^{\frac{1}{3}} + 2) = 0$$

$$4 \cdot x^{\frac{1}{3}} - 3 = 0 \quad \text{of} \quad x^{\frac{1}{3}} + 2 = 0$$

$$4\sqrt[3]{x} = 3 \quad \sqrt[3]{x} = -2$$

$$x^{-\frac{1}{2}} + 2 = 0$$

$$\sqrt[3]{x} = \frac{3}{4} \quad (\sqrt[3]{x})^3 = (-2)^3$$

$$(\sqrt{x})^3 = \left(\frac{3}{4}\right)^3 \quad x = -8$$

$$x = \frac{27}{64}$$

opl

$$21. \quad x - 5\sqrt{x} + 4 = 0$$

$$x - 5x^{\frac{1}{2}} + 4 = 0$$

$$(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 4) = 0$$

$$x^{\frac{1}{2}} - 1 = 0 \quad \text{of} \quad x^{\frac{1}{2}} - 4 = 0$$

$$\sqrt{x} = 1 \quad \sqrt{x} = 4$$

$$(\sqrt{x})^2 = 1^2 \quad (\sqrt{x})^2 = 4^2$$

$$x = 1 \quad x = 16$$

$$23. \quad x^{-1} - 7x^{-\frac{1}{2}} = 18$$

$$x^{-1} - 7x^{-\frac{1}{2}} - 18 = 0$$

$$(x^{-\frac{1}{2}} - 9)(x^{-\frac{1}{2}} + 2) = 0$$

$$x^{-\frac{1}{2}} - 9 = 0 \quad \text{of}$$

$$\frac{1}{x^{\frac{1}{2}}} = \frac{9}{1}$$

$$\frac{1}{x^{\frac{1}{2}}} = \frac{-2}{1}$$

$$9x^{\frac{1}{2}} = 1$$

$$-2x^{\frac{1}{2}} = 1$$

$$\sqrt{x} = \frac{1}{9}$$

$$\sqrt{x} = -\frac{1}{2}$$

$$(\sqrt{x})^2 = \left(\frac{1}{9}\right)^2$$

geen \mathbb{R}

$$x = \frac{1}{81}$$

Hoofstuk 1

Eksponentes en Wortel

Antwoorde 5: Gemeng

$$\begin{aligned}
 1.1 \quad & \frac{9^{\frac{3}{4}-3^{\frac{1}{2}}}}{4 \cdot 3^{\frac{1}{2}} - \sqrt{3}} \\
 &= \frac{\left(3^{\frac{2}{1}}\right)^{\frac{3}{4}-3^{\frac{1}{2}}}}{4 \cdot 3^{\frac{1}{2}} - 3^{\frac{1}{2}}} \\
 &= \frac{3^{\frac{3}{2}-3^{\frac{1}{2}}}}{4 \cdot 3^{\frac{1}{2}} - 3^{\frac{1}{2}}} \quad 3^{\frac{3}{2}} = 3^{1+\frac{1}{2}} = 3^1 \cdot 3^{\frac{1}{2}} \\
 &= \frac{3^1 \cdot 3^{\frac{1}{2}-3^{\frac{1}{2}}}}{4 \cdot 3^{\frac{1}{2}} - 3^{\frac{1}{2}}} \\
 &= \frac{3^{\frac{1}{2}}(3-1)}{3^{\frac{1}{2}}(4-1)} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad & \frac{x^n(x^n-2)-3}{x^{n+1}-3x} \\
 &= \frac{1 \cdot x^{2n}-2x^n-3}{x^n \cdot x-3x} \quad ax^2 + bx + c \\
 &= \frac{(x^n+1)(x^n-3)}{x(x^n-3)} \quad \therefore a \cdot x^{2n} + b \cdot x^n + c \\
 &= \frac{x^n+1}{x}
 \end{aligned}$$

$$\begin{aligned}
 1.3 \quad & \frac{a^{x+3}+2 \cdot a^{x+2}}{a^{x+2} \cdot a^{x-1}}; \quad a \neq 0; 2 \\
 &= \frac{a^3 a^x + 2a^2 a^x}{a^x + 2a^{-1} a^x} \\
 &= \frac{a^x a^2 (a+2)}{a^x (1+2a^{-1})} \quad 2 \cdot a^{-1} = 2 \cdot \frac{1}{a} = \frac{2}{1} \cdot \frac{1}{a} = \frac{2}{a} \\
 &= \frac{a^2(a+2)}{\frac{1}{1} + \frac{2}{a}} \\
 &= \frac{a^2(a+2)}{\frac{a+2}{a}} \\
 &= \frac{a^2(a+2)}{1} \times \frac{a}{(a+2)} \\
 &= a^3
 \end{aligned}$$

$$\begin{aligned}
 1.4 \quad & \left(\frac{1}{3^{-1}+2^{-1}}\right)^2 + (3^{-1} + 2^{-1})^{-2} \\
 &= \left(\frac{1}{\frac{1}{3}+\frac{1}{2}}\right)^2 + \left(\frac{1}{3} + \frac{1}{2}\right)^{-2} \\
 &= \left(\frac{1}{\frac{2+3}{6}}\right)^2 + \left(\frac{2+3}{6}\right)^{-2} \\
 &= \left(\frac{1}{\frac{5}{6}}\right)^2 + \left(\frac{5}{6}\right)^{-2} = \left(\frac{6}{5}\right)^2 + \frac{5^{-2}}{6^{-2}} = \frac{6^2}{5^2} + \frac{6^2}{5^2} = \frac{36}{25} + \frac{36}{25} = \frac{72}{25}
 \end{aligned}$$

$$\begin{aligned}
1.5 \quad & \sqrt{\frac{2^{n+3}+2^n}{2^{n-2}}} \\
&= \sqrt{\frac{2^3 2^n + 2^n}{2^{-2} 2^n}} \\
&= \sqrt{\frac{2^n(2^3+1)}{2^{-2} 2^n}} \\
&= \sqrt{\frac{(2^3+1)}{2^{-2}}} \\
&= \sqrt{\frac{9}{\frac{1}{2^2}}} = \sqrt{\frac{9}{\frac{1}{4}}} = \sqrt{9 \cdot 4} = \sqrt{36} = 6
\end{aligned}$$

$$\begin{aligned}
1.6 \quad & \frac{a^{2m-2}+a^{-2m}}{a^m-a^{-m}} \\
&= \frac{\frac{a^{2m}}{1} - \frac{2}{1} + \frac{1}{a^{2m}}}{\frac{a^m}{1} - \frac{1}{a^m}} \\
&= \frac{a^{2m} a^{2m} - 2 a^{2m+1}}{a^m a^m - 1} \\
&= \frac{1 a^{4m} - 2 a^{2m+1}}{a^{2m} - 1} \\
&= \frac{(a^{2m-1})(a^{2m-1})}{(a^{2m-1})} \\
&= \frac{(a^{2m-1})(a^{2m-1})}{a^{2m}} \times \frac{a^m}{(a^{2m-1})} \\
&= \frac{(a^{2m-1})}{a^m}
\end{aligned}$$

$$\begin{aligned}
1.7 \quad & \frac{(2 \cdot 4^2)^{\frac{1}{6}}}{27^{\frac{1}{3}}} \div \left(\frac{6^2}{8}\right)^{-\frac{5}{6}} \\
&= \frac{(2^1 \cdot 2^4)^{\frac{1}{6}}}{\left(3 \frac{3}{1}\right)^{\frac{1}{3}}} \div \left[\frac{(2 \cdot 3)^2}{2^3}\right]^{-\frac{5}{6}} \\
&= \frac{\left(2 \frac{5}{1}\right)^{\frac{1}{6}}}{3^1} \div \left[2^2 \cdot 3^2 \cdot 2^{-3}\right]^{-\frac{5}{6}} \\
&= 2^{\frac{5}{6}} \cdot 3^{-1} \div \left[2^2 \cdot 3^2 \cdot 2^{-3}\right]^{-\frac{5}{6}} \\
&= \frac{2^{\frac{5}{6}} \cdot 3^{-1}}{\left[2^2 \cdot 3^2 \cdot 2^{-3}\right]^{-\frac{5}{6}}} \\
&= \frac{2^{\frac{5}{6}} \cdot 3^{-1}}{\left[3 \frac{2}{1} 2 \frac{-1}{1}\right]^{-\frac{5}{6}}} \\
&= \frac{2^{\frac{5}{6}} \cdot 3^{-1}}{3^{-\frac{5}{3}} 2^{\frac{5}{6}}} \\
&= 2^{\frac{5}{6}} \cdot 3^{-1} \cdot 3^{\frac{5}{3}} 2^{-\frac{5}{6}} = 2^0 \cdot 3^{-1+\frac{5}{3}} = 1 \cdot 3^{\frac{2}{3}} = 3^{\frac{2}{3}}
\end{aligned}$$

$$\begin{aligned}
1.8 \quad & \frac{3^{2x+1}+9^x}{3^x \cdot 3^{x+1} - (3^x)^2} \\
&= \frac{3 \cdot 3^{2x} + 3^{2x}}{3^x \cdot 3^1 \cdot 3^x - 3^{2x}} \\
&= \frac{3^{2x}(3+1)}{3^{2x} \cdot 3 - 3^{2x}} \\
&= \frac{3^{2x} \cdot 4}{3^{2x}(3-1)} \\
&= \frac{4}{2} = 2
\end{aligned}$$

$$\begin{aligned}
1.9 \quad & \frac{a^{x+2} - 3a^{x+1}}{3a^{x-2} - a^{x-1}} \\
&= \frac{a^2 \cdot a^x - 3a^1 \cdot a^x}{3a^{-2} \cdot a^x - a^{-1} \cdot a^x} \\
&= \frac{a^x(a^2 - 3a)}{a^x(3a^{-2} - a^{-1})} \\
&= \frac{(a^2 - 3a)}{\left(\frac{3}{a^2} - \frac{1}{a^1}\right)} \\
&= \frac{(a^2 - 3a)}{\left(\frac{3-a}{a^2}\right)} \\
&= (a^2 - 3a) \times \frac{a^2}{(3-a)} \\
&= \frac{a(a-3)}{1} \times \frac{a^2}{-(a-3)} \\
&= -a^3
\end{aligned}$$

$$\begin{aligned}
1.10 \quad & \frac{x^{-1} + y^{-1}}{(x+y)^{-1}} \\
&= \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{(x+y)}} \\
&= \frac{\frac{(y+x)}{xy}}{\frac{1}{(x+y)}} \\
&= \frac{(x+y)}{xy} \times \frac{(x+y)}{1} \\
&= \frac{(x+y)^2}{xy}
\end{aligned}$$

$$\begin{aligned}
1.11 \quad & \left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)^2 - \left(a^{\frac{1}{2}} - a^{-\frac{1}{2}}\right)^2 \\
&= a^1 + 2 \cdot a^{\frac{1}{2}} \cdot a^{-\frac{1}{2}} + a^{-1} - \left(a^1 - 2 \cdot a^{\frac{1}{2}} \cdot a^{-\frac{1}{2}} + a^{-1}\right) \\
&= a^1 + 2 + a^{-1} - a^1 + 2 - a^{-1} \\
&= 4
\end{aligned}$$

$$\begin{aligned}
1.12 \quad & \frac{24^{\frac{1}{2}} \cdot 16^{\frac{1}{3}}}{54^{-\frac{1}{6}}} \\
&= \frac{(2^3 \cdot 3^1)^{\frac{1}{2}} \cdot (2^4)^{\frac{1}{3}}}{\left(2 \cdot 3^{\frac{3}{1}}\right)^{-\frac{1}{6}}} \\
&= \frac{2^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} \cdot 2^{\frac{4}{3}}}{2^{-\frac{1}{6}} \cdot 3^{-\frac{1}{2}}} \\
&= 2^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} \cdot 2^{\frac{4}{3}} \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{1}{2}} = 2^{\frac{3}{2} + \frac{4}{3} + \frac{1}{6}} \cdot 3^1 = 2^{\frac{9+8+1}{6}} \cdot 3 = 2^3 \cdot 3 = 8 \cdot 3 = 24
\end{aligned}$$

$$\begin{aligned}
1.13 \quad & (x-2+x^{-1})^{\frac{1}{2}} \cdot (x^{\frac{1}{2}} - x^{-\frac{1}{2}})^{-1} \\
&= \left(\frac{x}{1} - \frac{2}{1} + \frac{1}{x}\right)^{\frac{1}{2}} \cdot \left(\frac{x^{\frac{1}{2}}}{1} - \frac{1}{x^{\frac{1}{2}}}\right)^{-1} \\
&= \left(\frac{x \cdot x - 2 \cdot x + 1}{x}\right)^{\frac{1}{2}} \cdot \left(\frac{x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} - 1}{x^{\frac{1}{2}}}\right)^{-1} \\
&= \left(\frac{x^2 - 2x + 1}{x}\right)^{\frac{1}{2}} \cdot \frac{(x^1 - 1)^{-1}}{\left(x^{\frac{1}{2}}\right)^{-1}} \\
&= \frac{(x^2 - 2x + 1)^{\frac{1}{2}}}{x^{\frac{1}{2}}} \cdot \frac{x^{\frac{1}{2}}}{(x^1 - 1)} \\
&= \frac{[(x-1)(x-1)]^{\frac{1}{2}}}{x^{\frac{1}{2}}} \cdot \frac{x^{\frac{1}{2}}}{(x^1 - 1)} \\
&= \frac{\left[(x-1)^{\frac{2}{1}}\right]^{\frac{1}{2}}}{x^{\frac{1}{2}}} \cdot \frac{x^{\frac{1}{2}}}{(x-1)} = \frac{(x-1)}{x^{\frac{1}{2}}} \cdot \frac{x^{\frac{1}{2}}}{(x-1)} = 1
\end{aligned}$$

$$\begin{aligned}
1.14 \quad & \frac{3 \cdot 2^x - 4 \cdot 2^{x+2}}{2^x - 2^{x+1}} \\
&= \frac{3 \cdot 2^x - 4 \cdot 2^2 \cdot 2^x}{2^x - 2 \cdot 2^x} \\
&= \frac{2^x(3 - 4 \cdot 2^2)}{2^x(1 - 2)} \\
&= \frac{3 - 16}{-1} = \frac{-13}{-1} = 13
\end{aligned}$$

$$\begin{aligned}
1.15 \quad & 18^{\frac{1}{3}} \cdot 50^{-\frac{1}{3}} \cdot 12^{\frac{1}{3}} \cdot 20^{\frac{2}{3}} \\
&= (2 \cdot 3^2)^{\frac{1}{3}} \cdot (2 \cdot 5^2)^{-\frac{1}{3}} \cdot (2^2 \cdot 3)^{\frac{1}{3}} \cdot (2^2 \cdot 5)^{\frac{2}{3}} \\
&= 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} \cdot 2^{-\frac{1}{3}} \cdot 5^{-\frac{2}{3}} \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot 2^{\frac{4}{3}} \cdot 5^{\frac{2}{3}} \\
&= 2^{\frac{1}{3} - \frac{1}{3} + \frac{2}{3} + \frac{4}{3}} \cdot 3^{\frac{2}{3} + \frac{1}{3}} \cdot 5^{-\frac{2}{3} + \frac{2}{3}} \\
&= 2^2 3^1 5^0 = 4 \cdot 3 \cdot 1 = 12
\end{aligned}$$

$$\begin{aligned}
1.17 \quad & \sqrt{\sqrt{13} - 2} \cdot \sqrt{\sqrt{13} + 2} \\
&= \sqrt{(\sqrt{13} - 2)(\sqrt{13} + 2)} \\
&= \sqrt{13 - 4} = \sqrt{9} = 3
\end{aligned}$$

$$\begin{aligned}
1.16 \quad & \frac{4^{x-1} - 2^{2x+1}}{4^x} + 2 \\
&= \frac{(2^2)^{x-1} - 2^{2x+1}}{(2^2)^x} + 2 \\
&= \frac{2^{2x-2} - 2^{2x+1}}{2^{2x}} + 2 \\
&= \frac{2^{-2} \cdot 2^{2x} - 2^1 \cdot 2^{2x}}{2^{2x}} + 2 \\
&= \frac{2^{2x}(2^{-2} - 2^1)}{2^{2x}} + 2 \\
&= \frac{(2^{-2} - 2^1)}{1} + 2 \\
&= \frac{1}{2^2} - 2 + 2 \\
&= \frac{1}{4} - 2 + 2 = \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
1.18 \quad & \frac{\sqrt{p^2 - q^2} \times (p+q)^{\frac{5}{2}}}{(p-q)^{\frac{1}{2}}} \\
&= \frac{\sqrt{(p-q)(p+q)} \times (p+q)^{\frac{5}{2}}}{(p-q)^{\frac{1}{2}}} \\
&= \frac{(p-q)^{\frac{1}{2}} (p+q)^{\frac{1}{2}} \times (p+q)^{\frac{5}{2}}}{(p-q)^{\frac{1}{2}}} \\
&= (p+q)^{\frac{1}{2}} \times (p+q)^{\frac{5}{2}} \\
&= (p+q)^3
\end{aligned}$$

$$\begin{aligned}
1.19 \quad & \sqrt[3]{(1-\sqrt{2})^2} \cdot \sqrt[3]{3+2\sqrt{2}} \\
&= \sqrt[3]{1-2\sqrt{2}+2} \cdot \sqrt[3]{3+2\sqrt{2}} \\
&= \sqrt[3]{(3-2\sqrt{2})} \cdot \sqrt[3]{3+2\sqrt{2}} \\
&= \sqrt[3]{(3-2\sqrt{2}) \cdot (3+2\sqrt{2})} \\
&= \sqrt[3]{9-4 \cdot 2} = \sqrt[3]{1} = 1
\end{aligned}$$

$$\begin{aligned}
1.20 \quad & \frac{5^{2006}-5^{2004}+24}{5^{2004}+1} \\
&= \frac{5^{2004+2}-5^{2004}+24}{5^{2004}+1} \\
&= \frac{(5^{2004} \cdot 5^2 - 5^{2004}) + 24}{5^{2004}+1} \\
&= \frac{5^{2004}(5^2-1)+24}{5^{2004}+1} \\
&= \frac{5^{2004}(24)+24}{5^{2004}+1} \\
&= \frac{24 \cdot 5^{2004} + 24}{5^{2004}+1} \\
&= \frac{24(5^{2004}+1)}{(5^{2004}+1)} = 24
\end{aligned}$$

$$1.21 \quad \frac{a^2 \times \sqrt[3]{b}}{\sqrt{a} \times (-2b^3)^3} = k \times a^x \times b^y$$

$$\frac{a^2 \times b^{\frac{1}{3}}}{a^{\frac{1}{2}} \times (-2)^3 b^9} = k \times a^x \times b^y$$

$$-\frac{1}{8} \times a^2 \times b^{\frac{1}{3}} \times a^{-\frac{1}{2}} \times b^{-9} = k \times a^x \times b^y$$

$$-\frac{1}{8} \times a^{\frac{3}{2}} b^{-\frac{26}{3}} = k \times a^x \times b^y$$

$$\therefore k = -\frac{1}{8}; x = \frac{3}{2}; y = -\frac{26}{3}$$

$$1.22 \quad 3^{9x} = 64 \text{ en } 5^{\sqrt{p}} = 64$$

$$[(\sqrt{5})^2]^{\sqrt{p}} = 64 \text{ en } 3^{9x} = 64$$

$$(\sqrt{5})^{2\sqrt{p}} = 64 \quad \sqrt[3]{3^{9x}} = \sqrt[3]{64}$$

$$(\sqrt{5}^{\sqrt{p}})^2 = 64 \quad 3^{3x} = 4 \dots (2)$$

$$\sqrt{(\sqrt{5}^{\sqrt{p}})^2} = \sqrt{64}$$

$$\sqrt{5}^{\sqrt{p}} = 8 \dots (1)$$

$$\text{Dus is: } \frac{[3^{x-1}]^3}{\sqrt{5}^{\sqrt{p}}} = \frac{3^{3x-3}}{\sqrt{5}^{\sqrt{p}}}$$

$$= \frac{3^{3x} \cdot 3^{-3}}{\sqrt{5}^{\sqrt{p}}}$$

$$= \frac{4 \times 3^{-3}}{8} \text{ uit (1) en (2)}$$

$$= \frac{4 \times \frac{1}{27}}{8}$$

$$= \frac{1}{54}$$

$$1.23 \quad LK = \frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{8}}$$

$$RK = -2$$

$$= \frac{2}{1+\sqrt{2}} - \frac{8}{\sqrt{4 \cdot 2}}$$

$$= \frac{2}{1+\sqrt{2}} - \frac{8}{2\sqrt{2}}$$

$$= \frac{2}{1+\sqrt{2}} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})} - \frac{8}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} - \frac{8\sqrt{2}}{2 \cdot 2}$$

$$= \frac{2-2\sqrt{2}}{1-2} - \frac{8\sqrt{2}}{4}$$

$$= \frac{2-2\sqrt{2}}{-1} - \frac{2\sqrt{2}}{1}$$

$$= -2 + 2\sqrt{2} - 2\sqrt{2}$$

$$= -2 = RK$$

1.24 Bewys dat: $\frac{\sqrt{27}-\sqrt{12}}{\sqrt{75}-\sqrt{48}} = ab \cdot (ab)^{-1}$

$$\begin{aligned}
 LK &= \frac{\sqrt{27}-\sqrt{12}}{\sqrt{75}-\sqrt{48}} & RK &= ab \cdot (ab)^{-1} \\
 &= \frac{3\sqrt{3}-2\sqrt{3}}{5\sqrt{3}-4\sqrt{3}} & &= \frac{ab}{(ab)^1} \\
 &= \frac{\sqrt{3}}{\sqrt{3}} & &= 1 \\
 &= 1 & &= LK
 \end{aligned}$$

1.25 Herskryf die volgende uitdrukking as 'n mag van x : $\frac{x\sqrt{x\sqrt{x\sqrt{x}}}}{\sqrt[8]{x^7}}$

$$\begin{aligned}
 \frac{x\sqrt{x\sqrt{x\sqrt{x}}}}{\sqrt[8]{x^7}} &= \frac{x\sqrt{x\sqrt{x^1 \cdot x^{\frac{1}{2}}}}}{\sqrt[8]{x^7}} \\
 &= \frac{x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{x^{\frac{7}{8}}} \\
 &= \frac{x\sqrt{x^1 \cdot x^{\frac{3}{4}}}}{x^{\frac{7}{8}}} \\
 &= \frac{x\sqrt{x^{\frac{7}{4}}}}{x^{\frac{7}{8}}} \\
 &= \frac{x \cdot x^{\frac{7}{8}}}{x^{\frac{7}{8}}} \\
 &= x
 \end{aligned}$$

2.1 $2^6 \cdot 3^{x-1} = 3^4 \cdot 2^{x+1}$

$$2^6 \cdot 3^x \cdot 3^{-1} = 3^4 \cdot 2^x \cdot 2^1$$

$$\frac{2^6 \cdot 3^x \cdot 3^{-1}}{2^6 \cdot 3^{-1} \cdot 2^x} = \frac{3^4 \cdot 2^1 \cdot 2^1}{2^6 \cdot 3^{-1} \cdot 2^x}$$

$$\frac{3^x}{2^x} = \frac{3^4 \cdot 2^1}{2^6 \cdot 3^{-1}}$$

$$\left(\frac{3}{2}\right)^x = \frac{3^4 \cdot 3^1}{2^6 \cdot 2^{-1}}$$

$$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^5$$

$$x = 5$$

2.2 $2^{x-1} \sqrt{2^x} = \frac{1}{32}$

$$2^{x-1} 2^{\frac{x}{2}} = \frac{1}{2^5}$$

$$2^{x-1+\frac{x}{2}} = 2^{-5}$$

$$x-1+\frac{x}{2} = -5$$

$$x+\frac{x}{2} = -4$$

$$\frac{x}{1} \times \frac{2}{1} + \frac{x}{2} \times \frac{2}{1} = -4 \times \frac{2}{1}$$

$$2x+x = -8$$

$$3x = -8$$

$$x = -\frac{8}{3}$$

2.3 $2(x+1)^{\frac{3}{2}} = \frac{27}{4}$

$$\frac{2(x+1)^{\frac{3}{2}}}{2} = \frac{27}{4} \div \frac{2}{1}$$

$$(x+1)^{\frac{3}{2}} = \frac{27}{4} \times \frac{1}{2}$$

$$(x+1)^{\frac{3}{2}} = \frac{27}{8}$$

$$\sqrt{(x+1)^3} = \frac{3^3}{2^3}$$

$$\left(\sqrt{(x+1)^3}\right)^2 = \left(\frac{3^3}{2^3}\right)^2$$

$$(x+1)^3 = \frac{3^6}{2^6}$$

$$\sqrt[3]{(x+1)^3} = \sqrt[3]{\frac{3^6}{2^6}}$$

$$x+1 = \frac{3^2}{2^2}$$

$$x = \frac{9}{4} - 1$$

$$x = \frac{5}{4}$$

$$2.4 \quad (0,5)^{x^2-9} = 4^{x+3}$$

$$\left(\frac{1}{2}\right)^{x^2-9} = (2^2)^{x+3}$$

$$(2^{-1})^{x^2-9} = 2^{2x+6}$$

$$2^{-x^2+9} = 2^{2x+6}$$

$$-x^2 + 9 = 2x + 6$$

$$-x^2 - 2x + 3 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3 \quad \text{of} \quad x = 1$$

$$2.6 \quad 3^{x+1} + 5 \cdot 3^{x-1} - 42 = 0$$

$$3^{x+1} + 5 \cdot 3^{x-1} = 42$$

$$3^x \cdot 3^1 + 5 \cdot 3^x \cdot 3^{-1} = 42$$

$$3^x(3 + 5 \cdot 3^{-1}) = 42$$

$$3^x\left(3 + \frac{5}{3}\right) = 42$$

$$3^x\left(\frac{14}{3}\right) = 42$$

$$\frac{3^x\left(\frac{14}{3}\right)}{\left(\frac{14}{3}\right)} = \frac{42}{\left(\frac{14}{3}\right)}$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x = 2$$

$$2.8 \quad x^{\frac{1}{2}} - 2x^{\frac{1}{4}} - 3 = 0$$

$$\left(x^{\frac{1}{4}} - 3\right)\left(x^{\frac{1}{4}} + 1\right) = 0$$

$$x^{\frac{1}{4}} - 3 = 0 \quad \text{of} \quad x^{\frac{1}{4}} + 1 = 0$$

$$\sqrt[4]{x} = 3 \quad \sqrt[4]{x} = -1$$

$$(\sqrt[4]{x})^4 = 3^4 \quad \text{geen } \mathbb{R} \text{ opl}$$

$$x = 81$$

$$2.5 \quad 5 \cdot \sqrt[3]{x^2} - 20 = 0$$

$$5 \cdot \sqrt[3]{x^2} = 20$$

$$\sqrt[3]{x^2} = 4$$

$$\left(\sqrt[3]{x^2}\right)^3 = 4^3$$

$$x^2 = 64$$

$$\sqrt{x^2} = \pm \sqrt{64}$$

$$x = \pm 8$$

$$2.7 \quad 9^{x+1} - 10 \cdot 3^x + 1 = 0$$

$$(3^2)^{x+1} - 10 \cdot 3^x + 1 = 0$$

$$3^{2x+2} - 10 \cdot 3^x + 1 = 0$$

$$3^{2x} \cdot 3^2 - 10 \cdot 3^x + 1 = 0$$

$$9 \cdot 3^{2x} - 10 \cdot 3^x + 1 = 0$$

$$(9 \cdot 3^x - 1)(1 \cdot 3^x - 1) = 0$$

$$9 \cdot 3^x = 1 \quad \text{of} \quad 1 \cdot 3^x = 1$$

$$3^x = \frac{1}{9} \quad \text{of} \quad 3^x = 3^0$$

$$3^x = 3^{-2} \quad \text{of} \quad x = 0$$

$$x = -2$$

2.9 $3^{x+2} - 10 = -3^{-x}$ nie gemeenskaplike faktor - moet kwadratiese 3 term wees

$$3^x \cdot 3^2 - 10 = -1 \cdot 3^{-x} \quad \text{split eksponente sodat jy kan faktoriseer}$$

$$3^x \cdot 3^2 - 10 = \frac{-1}{3^x} \quad \text{maak negatiewe eksponente positief}$$

$$3^x \cdot 3^2 \times 3^x - 10 \times 3^x = \frac{-1}{3^x} \times \frac{3^x}{1} \quad \text{verwyder breuke}$$

$$3^2 \times 3^{2x} - 10 \cdot 3^x = -1 \quad \text{kwadratiese 3 term - kry standaardvorm}$$

$$9 \cdot 3^{2x} - 10 \cdot 3^x + 1 = 0 \quad \text{faktoriseer soos } 9y^2 - 10y + 1 = 0$$

$$(9 \cdot 3^x - 1)(1 \cdot 3^x - 1) = 0 \quad (9y - 1)(1y - 1) = 0$$

$$9 \cdot 3^x = 1 \quad \text{of} \quad 3^x = 1$$

$$\frac{9 \cdot 3^x}{9} = \frac{1}{9} \quad 3^x = 3^0$$

$$3^x = \frac{1}{9} \quad x = 0$$

$$3^x = 3^{-2}$$

$$x = -2$$

2.10

$$2^{x+2} + 7\sqrt{2^x} = 2 \quad \text{skryf wortelvorm oor in magsvorm}$$

$$2^x \cdot 2^2 + 7 \cdot 2^{\frac{x}{2}} - 2 = 0 \quad \text{kwadratiese 3 term want die } x \text{ eksponent}$$

$$4 \cdot 2^x + 7 \cdot 2^{\frac{x}{2}} - 2 = 0 \quad \text{is dubbel die } \frac{x}{2} \text{ eksponent}$$

$$(4 \cdot 2^{\frac{x}{2}} - 1)(1 \cdot 2^{\frac{x}{2}} + 2) = 0 \quad \text{faktoriseer}$$

$$4 \cdot 2^{\frac{x}{2}} = 1 \quad \text{of} \quad 2^{\frac{x}{2}} = -2$$

$$2^{\frac{x}{2}} = \frac{1}{4} \quad \text{geen } \mathbb{R} \text{ oplossing}$$

$$2^{\frac{x}{2}} = 2^{-2}$$

$$\frac{x}{2} = \frac{-2}{1}$$

$$x = -4$$

Meer oor “Wiskunde Anibrand Antwoordboek Graad 11” en die outeur.

Ek is reeds vir 28 jaar betrokke by Wiskunde-onderrig vir graad 8 tot graad 12 leerders. Die afgelope 10 jaar is ek verbonde aan Hoërskool Die Wilgers in Pretoria, waar ek ‘n Wiskunde Akademie bedryf met een groep in elke graad.

Met die aanvang van die nuwe KABV sillabus in 2007 het ek begin om al my notas vir my Wiskunde-onderrig elektronies saam te stel met behulp van innoverende sagteware sodat dit alle onderwerpe met grafika en voorbeelde volledig verduidelik. Die graad 11 Wiskunde Anibrand Antwoordboek bied volledig uitgewerkte, verduidelikende antwoorde vir al die huiswerk probleme in die graad 11 “Wiskunde Anibrand Notaboek”.

Leerders wat hierdie boek gebruik om hulle huiswerk probleme te merk, kan uit die antwoorde self sien waar hulle gefouteer het en dan hulle foute korrigeer.

Ek gebruik hierdie antwoorde die afgelope 5 jaar in my klasaanbieding vir die graad 11 leerders. Dit stel my in staat om die antwoorde konstant te verbeter, soos wat ek dit in die klassituasie as nodig ervaar.

Die volledige antwoorde op huiswerkprobleme help leerders om selfvertroue in die vak te kry want hulle kan hulle probleme identifiseer wanneer hulle huiswerk doen, dit uitsorteer en dan voortgaan met die res van die huiswerk probleme.

Hierdie boek, saam met die Notaboek, is die antwoord vir alle graad 11 leerders wat wil presteer in Wiskunde en ook vir alle Wiskunde onderwysers wat sonder moeite ‘n kwaliteit Wiskunde klasaanbieding vir leerders wil bied.

www.wiskundeanibrand.com